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THE UNIVERSITY OF ALBERTA

OPERATION AND MAINTENANCE OF DISTRIBUTION NETWORKS

BY

© KUA HUNG KWENG

A THESIS
SUBMITTED TO FACULTY OF GRADUATE STUDIES AND RESEARCH IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

DEPARTMENT OF ELECTRICAL ENGINEERING

EDMONTON, ALBERTA
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(Supervisor)

Alan Koval
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Date : 10 April 1989

DEDICATION

To my parents

ABSTRACT

As utilities continuously accommodate consumer load demands, the network service standards (i.e., quality and continuity of service) of the electrical power utilities can be significantly affected by the changing loads. High quality and continuity of supply can only be met through adequate system planning. A major part of this planning process is the assessment of network reliability levels and individual customer service reliability levels which are affected by network loading configurations, equipment aging, and other factors such as maintenance activities and transient outages. Another major planning function is the assessment of the network load flow characteristics (e.g., voltage and current levels) which have a direct bearing on the network reliability levels. This thesis presents two algorithms that deal with these two major areas of power system planning.

The first computer algorithm developed in this thesis evaluates the reliability levels (i.e., continuity of service) at different load points within a given power system configuration. The algorithm uses the Weibull probability distribution, representing the life distribution of the network components, to evaluate the reliability indices over a given operating period. The second computer algorithm developed models a three-phase distribution network for operational characteristics. This algorithm provides a three-phase load flow procedure that models the unbalanced characteristics of a network.

The two computer programs proposed aid power utilities to analyze in detail the operational characteristics (e.g., load flow) and reliability levels of their designs so as to provide continuity and high quality of service. That is, the results of these studies enable the power utilities to improve their service to their customers more efficiently and economically.

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CHAPTER I INTRODUCTION

1.1 Introduction

A typical power system configuration is shown in Figure 1.1 below.

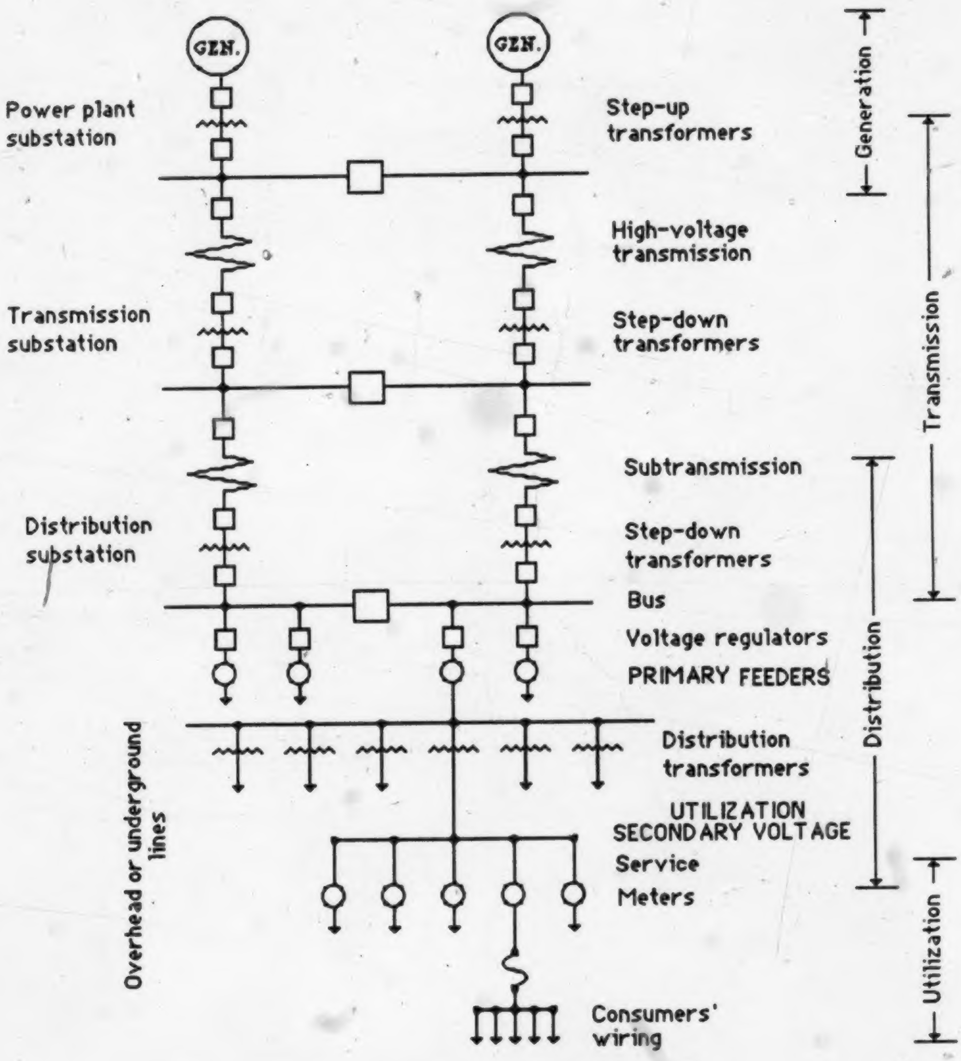

GEN.
GEN.
Power plant substation
Step-up transformers
High-voltage transmission
Transmission substation
Step-down transformers
Subtransmission
Distribution substation
Step-down transformers
Bus
Voltage regulators
PRIMARY FEEDERS
Distribution transformers
UTILIZATION
SECONDARY VOLTAGE
Service
Meters
Consumers' wiring
Overhead or underground lines
Generation
Transmission
Distribution
Utilization

Figure 1.1 A typical electrical power system.

The main purpose of an electrical power system is to efficiently generate, transmit, and distribute electric energy. The operations involve geographically dispersed and functionally complex monitoring and control systems, as illustrated in Figure 1.2. As seen in the figure, the energy management system (EMS) controls the overall system. The supervisory control and data acquisition (SCADA) system involves generation and transmission systems, while the distribution automation and control (DAC) system oversees the distribution system.

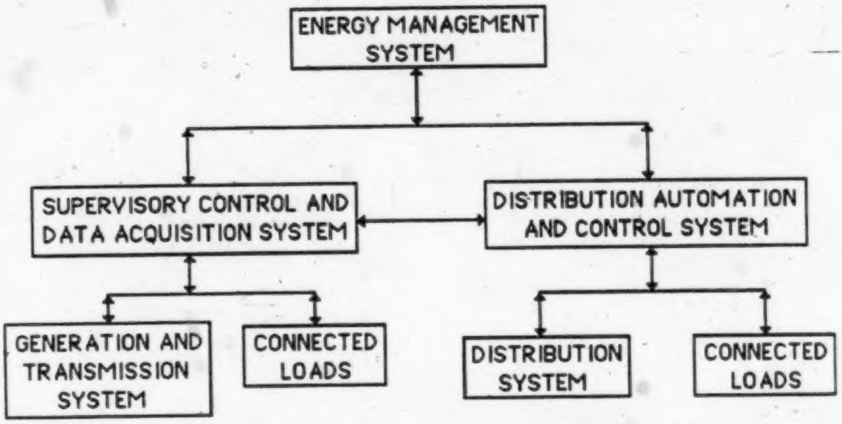

ENERGY MANAGEMENT SYSTEM
SUPERVISORY CONTROL AND DATA ACQUISITION SYSTEM
DISTRIBUTION AUTOMATION AND CONTROL SYSTEM
GENERATION AND TRANSMISSION SYSTEM
CONNECTED LOADS
DISTRIBUTION SYSTEM
CONNECTED LOADS

Figure 1.2 Monitoring and controlling an electrical power system.

With expansion in the use of electricity, demands on distribution systems have become greater and more complex. They not only have to serve greater numbers of consumers, but must supply greater individual loads that require closer supervision of voltage variations at the consumers' terminals. In addition, consumers demand increased reliability in their service with fewer interruptions of shorter duration than in the past because of their use of "sensitive" computer technology.

Therefore, the monitoring objectives of the DAC system are [1] :

(i) " improve the overall system efficiency in the usage of both capital and energy ;

(ii) increase market penetration of energy source ;

(iii) reduce reserve requirements in transmission and generation ; and

(iv) increase the level of reliability service to essential loads. "

1.2 Reliability Analysis

Considerable attention has been devoted by power utilities to the evaluation of the effectiveness of their systems from a reliability viewpoint. The bulk of the work has been done in the field of generation capacity reliability evaluation. During the last decade, more attention has been focussed on the reliability studies of transmission and distribution schemes [2-6]. One of the main concerns has been the development of accurate and consistent models to represent component and system variables. These variables and their associated parameters significantly affect the continuity of electrical service as seen by a utility's customers. In most publications [1,3,6-11], the failure processes are assumed to be characterized by random variables which can be mathematically described by a known probability density function (e.g., the most common assumption being the exponential distribution characterized by a constant failure rate.)

In this research, a reliability study of a network whose component failure and renewal processes are characterized by a Weibull statistical distribution is presented. This distribution allows the rate parameters to be variable with operational time as opposed to the classical models which assume a constant value. A detailed discussion of the significant impact of the use of the Weibull statistical distribution instead of the normally assumed exponential distribution in power system reliability analysis is presented.

Chapter II describes the reliability methodology proposed and discusses the results obtained when applying the proposed algorithm to the Standard IEEE 14 bus test network over a one year period, with the consideration of scheduled preventive maintenance outages and randomly occurring transient outages. The active, stuck breakers and common mode failure events overlapping the passive failure event were also investigated in detail using the Weibull statistical distribution. The effects of adverse weather and the possibility of isolating failed components from the network were also considered.

1.3 Unbalanced Three-Phase Load Flow

In the last few decades, large power utilities have used computer modelling methods to simulate the three-phase portion of their systems. Their methods are based on a balanced three-phase system assuming transposed lines and balanced loads. However, one of the problems encountered in power system operation is the generation of unbalanced voltages and currents in distribution and transmission lines with few or no transpositions, including unbalances arising in source and load connections. Distribution system loads are unbalanced (i.e., load connections) because the majority of the loads they serve are single phase. The presence of negative-sequence currents due to the unbalanced operating conditions at the generator terminals may overheat the rotor of the generator. In addition, because of unpredictable current distributions, protective relays can malfunction and active power loss in the transmission lines could be comparatively higher than expected because of parallel untransposed transmission lines. The unbalanced effects of these lines and the resulting unbalanced loads (i.e., a situation emphasized by unbalanced line impedances) must be analyzed in detail for power system planning.

To investigate these unbalance effects in any detail, a three-phase load flow solution that allows representation of all possible unbalances as they exist in the power systems without making any assumptions (such as symmetrical components and transposition) is essential. An algorithm using the Gauss-Seidel iteration method is presented. It forms the basis of a computer program developed for the specific purpose of solving the three-phase load flow problem. Due to the limitation of memory in the computer available at the University of Alberta, the Newton-Raphson method was not used, as the Jacobian Matrix of this method requires excessive memory for large systems. Even with the use of sparsity techniques, the memory available was not sufficient for studying a large power system. Hence, the Gauss-Seidel iteration method was used with a trade off in the number of iterations required for convergence.

The algorithm considers all the major configurations of transformer connections, such as delta-delta, delta-wye, wye-delta, and wye-wye. Grounded or ungrounded wye connections are also considered. The equivalent Pi-network of the transmission and distribution lines are used in the algorithm. The mutual coupling terms between the phases of the lines and the neutral and ground wires were also included in the study.

Chapter III illustrates and discusses the three-phase unbalanced load flow algorithm and its application to a test system.

1.4 Summary

In conclusion, this thesis proposes two computer programs that will assist the power utilities in analyzing in detail the operational characteristics (e.g., loading configuration) and the reliability levels (e.g., frequency of interruptions) of their new designs to provide a continuity and high quality of service. These algorithms are the basis of many other computer programs (e.g., fault, protection-coordination, switching transients, etc.) that are necessary in distribution system planning.

CHAPTER II RELIABILITY ANALYSIS

2.1 Introduction

A power system's main function is to supply consumers with electrical energy as economically and as reliably as possible by making economical use of the available operating system and apparatus. Here, the term reliable electrical service has customarily meant meeting the consumers' electrical energy requirements as demanded, and providing "continuity" of service consistent with the safety of personnel and equipment. On the other hand, quality electrical service involves meeting the consumer demand within specified voltage and frequency limits. To maintain reliable service to consumers, the utility must have adequate redundancy in its system to prevent a component outage from becoming a service interruption to the consumers. The cost of reliability to a utility's customers is dependent upon the rate at which their interruptions occur and the duration of these interruptions.

Basically three reliability indices, namely, the frequency of customer interruptions, the average duration of interruption and the expected annual duration of customer interruptions are used to characterize the continuity of a utility's electrical service. The basic data acquired by utilities in the form of outage reports is processed to yield customer service reliability indices [12]. If the service reliability levels are below customer expected levels and significant losses are incurred by them (e.g., commercial and industrial customers), then both the customer and the utility must assess the reliability cost - reliability worth [13-14] of the present electrical energy delivery system. Thus, a major problem facing utilities is to deliver electrical energy to its customers as economically as possible and provide an adequate assurance of continuity and quality of service. Prior to assessing the cost of service interruptions, it is necessary for a utility to quantitatively evaluate service reliability levels delivered by the utility's network configurations assessing the impact of alternative network operating configurations and various practices (e.g., maintenance, fault location, equipment redundancy, protection practices, etc.).

One of the difficulties in reliability modelling of power system network configurations occurs in characterizing the variables describing a network element's failure, maintenance and renewal processes. These variables and their associated parameters significantly affect the continuity of electrical services as seen by the utility's customers. In most studies [7-11], the failure processes are assumed to be random variables which can be mathematically described by a known probability density function (e.g., the most common assumption being the exponential distribution characterized by a constant failure rate; i.e., they assume that the failure and renewal processes are independent of time or constant). Similarly, the repair and maintenance processes are assumed to be described by the exponential distribution. Service reliability results based on the exponential distribution are usually expressed as long-term values assuming the network operating configuration does not change.

The use of the exponential distribution to characterize a network's failure, maintenance and renewal processes can be quite problematic and misleading. For example, if the network's electrical equipment is old, its failure rate may increase with in-service life and be significantly higher than the average failure rate. When *average* failure rates are used in a reliability model of an old electrical network, the frequency of interruptions will be significantly under-estimated. The converse is also true for modelling relatively new electrical systems whose failure rates are often significantly lower than the average value. Maintenance activities are scheduled at predefined times during the year to improve equipment reliability performance. Many reliability models cannot accommodate this process in their models [7-10,15].

In general, electrical equipments' hazard rates can be described by a bathtub curve, i.e., the failure rate changes with operating time. The exponential distribution, which gives average rate, has always been used to describe the failure rate of electrical equipment. The life performance of the equipment is therefore characterized by a moving average failure rate (λ_{ave}), i.e., a step-wise function based on past performance. The Weibull distribution on the other hand, gives the rate of change of the rate parameters, that is, based on past performance the equipment hazard rate can be approximated by :

$$K_1 \lambda_{ave} + K_2 \partial\lambda / \partial t$$

Therefore, the Weibull statistical distribution gives a more accurate approximation to the bathtub curve. The component past performance gives the shape and scale parameters that are necessary for the reliability forecast of a network configuration whose failure and renewal processes are based on a Weibull statistical distribution.

A knowledge of the network and individual consumer service reliability levels is an extremely important factor in the design of a utility's and consumer's electrical and electronic systems. Many reliability techniques simply evaluate the overall power system network reliability levels with very little attention being directed to evaluating consumer levels of reliability. However, from a consumer's viewpoint, their location within a power system network configuration has a significant impact on the level of service reliability they receive and may be quite different from the overall network system reliability levels of the power system network serving them. This thesis proposes the use of Weibull statistical distribution for reliability study of power system whose element (i.e., equipment) failure and renewal processes are age dependent. This distribution allows the rate parameters associated with the distribution to be variable as opposed to the exponential distribution which assumes a constant value. The reliability techniques used in this research are based on the minimum cut-set approach and the flow graph technique that allows the determination of individual consumer service reliability level [16-17].

Traditionally, reliability levels of complex power system network configurations have been evaluated assuming the underlying life distributions of electrical equipment can be represented by time invariant rates based on the exponential density function (i.e., a stationary process). This Chapter presents the results of a reliability study of the IEEE 14 bus network configuration where the network electrical equipment failure and renewal processes are represented by the Weibull statistical probability density functions (e.g., p.d.f.) where the equipment failure and repair rates are variable (e.g., a process associated with aging electrical equipment). The Weibull p.d.f. allows the rate parameters to vary with the operating life cycles of the network's electrical equipment. The impact of scheduled maintenance activities at predefined time intervals in a year is also presented in the thesis. A detailed discussion of the significant impact of the use of the Weibull statistical distribution instead of the normally assumed exponential distribution in power system reliability analysis is presented.

2.2 Weibull Distribution

The basic statistical properties of the Weibull distribution used in the thesis are summarized in this section. Further discussions of the distribution and a detailed treatment of the graphical methodology required to estimate the reliability or the Weibull shape and scale parameters are presented in Appendix A.

2.2.1 *Cumulative Density Function*

The three parameter Weibull cumulative or failure density distribution function (c.d.f.) for a random variable *t* is given by [18] :

$$F(t;\alpha,\beta,\delta) = 1 - \exp\left(-\frac{t-\delta}{\alpha-\delta}\right)^{\beta} \qquad ; \quad t \geq \delta \tag{2.1}$$

where :

t = operating time
α = scale parameter
β = shape parameter
δ = location parameter or minimum life

The two-parameter Weibull c.d.f. has a minimum life parameter of zero, and the failure distribution function is given by :

$$F(t;\alpha,\beta) = 1 - \exp\left(-\frac{t}{\alpha}\right)^{\beta} \qquad ; \quad t \geq 0 \tag{2.2}$$

Since the three-parameter distribution can always be converted to the two-parameter distribution by a simple linear transformation, the two parameter Weibull will be used to illustrate the properties of the distribution.

2.2.2 *Failure Density Function*

The failure density function of the Weibull distribution is defined as

$$f(t) = \frac{\beta(t - \delta)^{\beta-1}}{(\alpha - \delta)^{\beta}} \; \exp\left[-\left(\frac{t-\delta}{\alpha-\delta}\right)^{\beta}\right] \tag{2.3}$$

where :

δ (location parameter or minimum life) ≥ 0 ;
β (shape parameter) > 0 ;
α (scale parameter) > 0 ; and
$t \geq \delta \geq 0$.

Taking the minimum life, δ as zero,

$$f(t) = \frac{\beta\, t^{\beta-1}}{\alpha^{\beta}} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \tag{2.4}$$

The survivor or availability function, R(t), and the cumulative failure distribution, Q(t), are defined as :

$$R(t) = \int_0^{\infty} f(t)\, dt = 1 - F(t) \tag{2.5}$$

$$Q(t) = 1 - R(t) \tag{2.6}$$

where F(t) is the cumulative density function for the variable t. The failure or hazard rate is given as :

$$\lambda(t) = h(t) = \frac{f(t)}{R(t)} \tag{2.7}$$

The survivor function and the failure rate of the Weibull distribution can therefore be written as :

$$R(t) = \exp\left(-\frac{t}{\alpha}\right)^{\beta} \tag{2.8}$$

$$\lambda(t) = \frac{\beta\, t^{\beta-1}}{\alpha^{\beta}} \tag{2.9}$$

There are two special cases that can be deduced from the Weibull distribution, the first is when the shape parameter, $\beta = 1$ and the second when $\beta = 2$.

(a) *For $\beta = 1$*

In this case, equations (2.4) and (2.9) reduce to :

$$f(t) = \frac{1}{\alpha} \exp[-\frac{t}{\alpha}]$$

$$\lambda(t) = \frac{1}{\alpha}$$

If $\alpha = 1/\lambda$ then $\lambda(t) = \lambda$ which is constant, and these equations are identical to those for the exponential distribution, that is, the value of α represents the mean time to failure (MTTF).

(b) *For $\beta = 2$*

In this case, equations (2.4) and (2.9) reduce to :

$$f(t) = \frac{2t}{\alpha^2} \exp[-\frac{t^2}{\alpha^2}]$$

$$\lambda(t) = \frac{2t}{\alpha^2}$$

The two equations above are identical to those for the Rayleigh distribution.

It can be concluded from these two cases that the Weibull distribution can approximate a number of statistical distributions. This is a characteristic of the Weibull function, that it can be scaled and shaped by varying its shaping parameters. Therefore, the most significant parameter of the Weibull distribution is its shape parameter β which significantly determines the dispersion of the distribution from the expected value of the distribution as illustrated in Figure 2.1. As shown above, when the shape parameter is equal to one, the Weibull distribution reduces to the classical exponential distribution. As the shape parameter increases, the distribution becomes more skewed to the right. This factor will have a significant impact on calculated service reliability levels. Typical shapes that can be produced for the Weibull distribution are shown in Figure 2.1 for the failure density function, cumulative failure function and hazard rate. It is evident from Figure 2.1 that [10] :

$\beta < 1$ represents a decreasing failure rate or the debugging period ;

$\beta = 1$ represents a constant failure rate or the normal life period ; and

$\beta > 1$ represents an increasing failure rate or the wear out period.

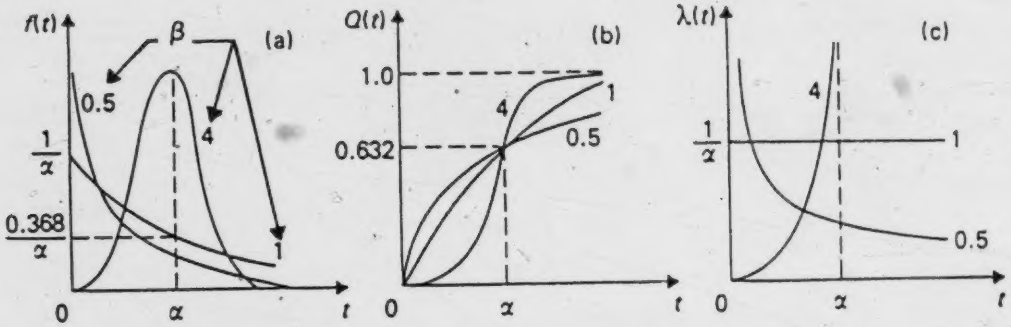


Figure 2.1 Weibull reliability functions. (a) Failure density function. (b) Cumulative failure distribution. (c) Failure rate. Parameters = values of β.

2.2.3 Maintainability Function

The maintainability function L(t) denotes the probability that when the repair begins at t = 0, it will be accomplished within time t.

The maintainability function, L(t), is defined as :

$$L(t) = \int_0^t f(y)\,dy \tag{2.10}$$

where :

t - is the predefined duration of repair

y - is the variable repair time

f(y) - is the probability density function of the duration of repair times

The repair rate function is expressed as :

$$\mu_m(t) = \frac{f(t)}{1 - L(t)} \tag{2.11}$$

where $\mu_m(t)$ is the time dependent repair rate function.

Therefore, for the Weibull distribution, the maintainability function and the repair rate function are defined as follows :

$$L(t) = 1 - \exp\left(-\frac{t}{\alpha}\right)^{\beta} \tag{2.12}$$

$$\mu_m(t) = \frac{\beta\, t^{\beta-1}}{\alpha^{\beta}} \tag{2.13}$$

where α and β are the scale and shape parameters, respectively, as defined previously.

2.2.4 Time Dependent Probability

In any two state model (i.e., an operating state and a failed state) the probabilities of being in the operating state and failed state, $P_0(t)$ and $P_1(t)$ respectively, based on the underlying distribution being characterized by the exponential distribution as a function of time, given that the system started in the operating state at time t = 0 are :

$$P_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda\, e^{-(\lambda+\mu)t}}{\lambda + \mu} \tag{2.14}$$

$$P_1(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda e^{-(\lambda+\mu)t}}{\lambda + \mu} \tag{2.15}$$

Therefore, assuming that the Weibull distribution scale and shape parameters of the elements within a network configuration for a particular operating time period are known (i.e., from equipment testing or past experience), then the reliability indices (e.g., frequency of failure) of the consumers can be calculated from equations (2.4), (2.13), (2.14), (2.15) based on the minimum-cut set approach [17].

2.3 Case Studies

A flow graph technique [17,19] is used to determine all the possible operational paths within a given power system network configuration that are required to maintain consumer continuity of service. The reliability methodology is based on the component life and renewal processes being represented by the Weibull distribution. A "critical" consumer location within the network configuration is selected and the reliability levels evaluated.

2.3.1 Reliability Modelling Assumptions

The IEEE 14 bus network configuration is used to demonstrate the significant changes in a customer's reliability levels for various assumed statistical distributions representing the failure and renewal processes of a network's electrical equipment and is shown in Figure 2.2 The single line diagram presented in this form does not show the network protective elements. However, in many reliability studies, it is implicitly assumed that when an outage or a combination of outages occurs in a network configuration, the electrical equipment is protected by the protecting elements which isolate faulty equipment from the active operating network configuration in an attempt to maintain continuity of service to as many customers as possible.

The following types of electrical equipment outages are usually considered in most reliability studies [8-10,15-18] :

1. permanent
2. temporary (i.e., often called transient)
3. maintenance.

Maintenance activities (i.e., forced and scheduled) are included in many reliability models and are subdivided according to the weather conditions at the time of equipment outages as follows :

1. normal weather periods
2. adverse weather periods - repair and restoration activities undertaken
3. adverse weather periods - repair and restoration activities discontinued.

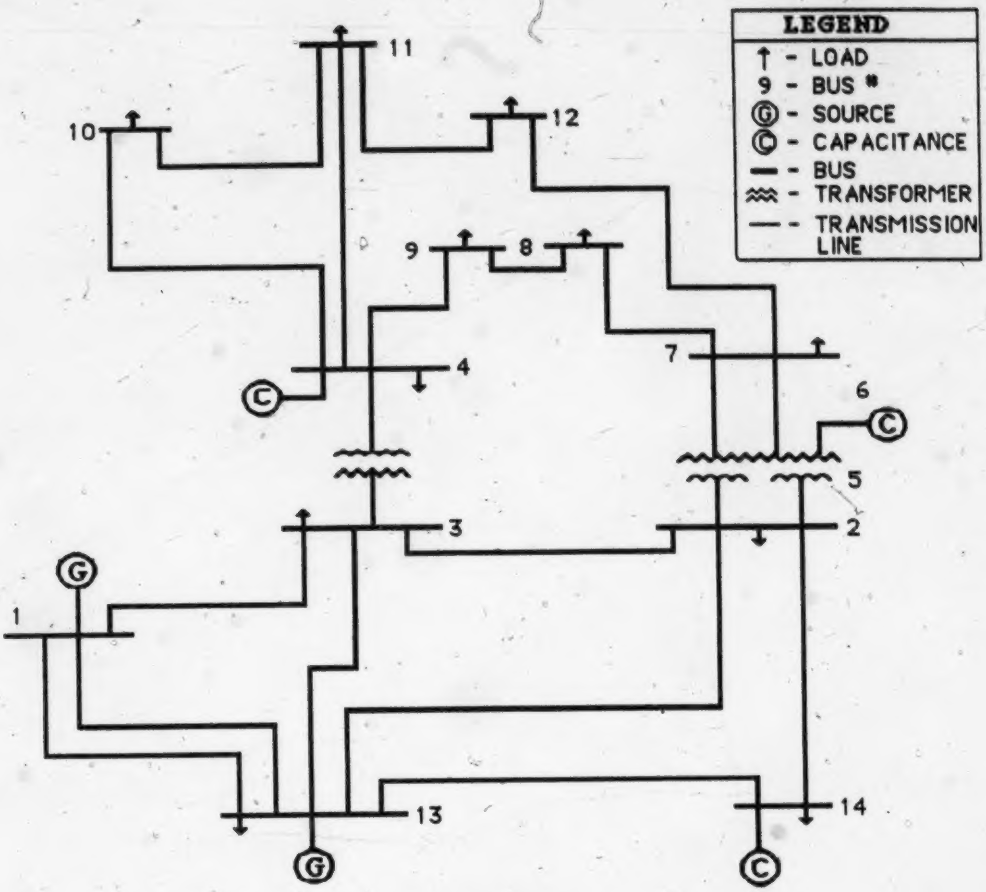

LEGEND
↑ - LOAD
9 - BUS #
Ⓖ - SOURCE
Ⓒ - CAPACITANCE
— - BUS
≈ - TRANSFORMER
— - TRANSMISSION LINE
1
2
3
4
5
6
7
8
9
10
11
12
13
14
G
C

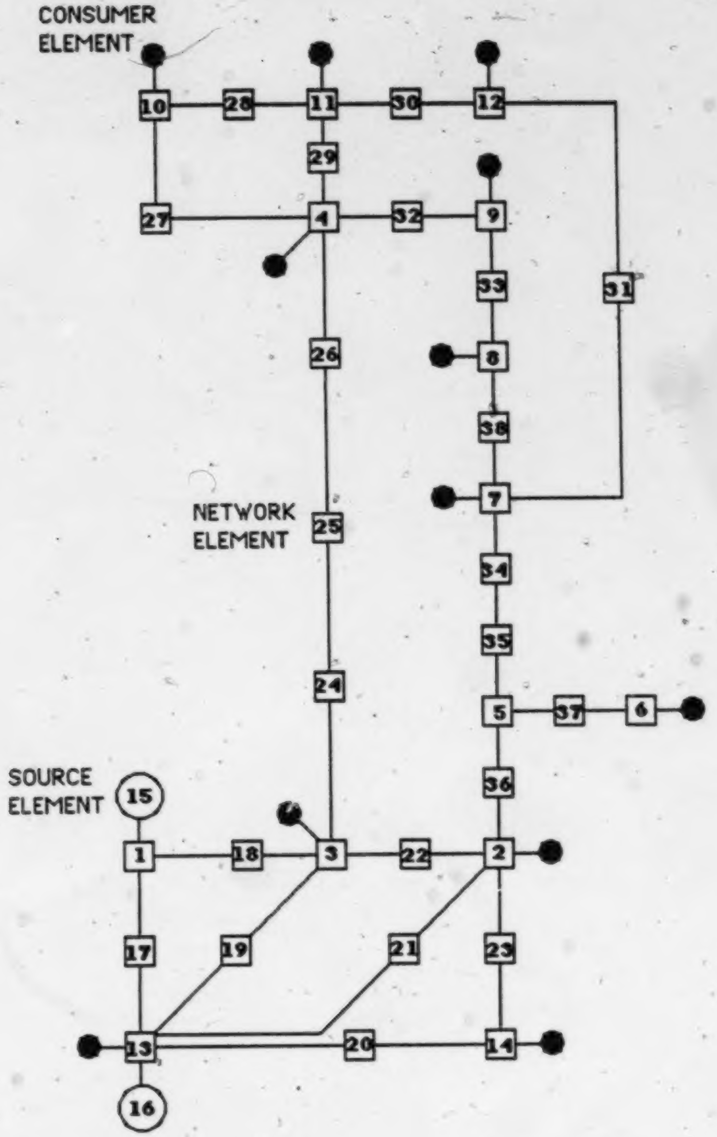
Figure 2.2 IEEE 14 bus network configuration.

In most reliability studies, the reliability levels calculated are called expected values or long term steady state solutions of the evolution equations describing the failure and renewal processes of the network. Short term studies require a significant modification of the basic reliability equations and the efforts are computationally very difficult and time consuming. In this study variable rate parameters are assumed for the network's equipment failure and renewal processes and the reliability results presented for various operating periods. By the use of the Weibull distribution, selected activities can be included in the model (e.g., scheduled maintenance activities which are defined during certain time periods in the year) to assess the impact and timing of these activities on customer service reliability levels. The difference in the reliability indices for various shape parameters are also investigated. Note that the value for the shape parameter (β) is not necessarily the same for all components. The shape parameters are obtained from equipment testing or from past experience of the component's characteristics, i.e., the same type of component may have the same shape parameter or they may be different depending on their environment and the stress level they are exposed to. Appendix A shows how the shape parameter can be obtained.

2.3.2 *Network Representation*

Prior to evaluating the service reliability levels being delivered to a particular customer's service entrance, it is convenient to convert the single line diagram of Figure 2.2 into an element diagram [17] as shown in Figure 2.3 where all transmission lines, bus connections and customer nodes are renumbered as element numbers. An element of a network is an electrical component whose failure will result in a discontinuity of the network operating paths. A network "operational path" is simply a set of elements interconnecting the source or generation elements through various electrical equipment elements to customer locations (i.e., load points).

The element diagram is the basis of many reliability models and provides a means of establishing the operational paths necessary to maintain continuity of service to any customer service location. When network elements fail individually or in various combinations, the network protective equipment isolates the faulty element(s) and alters the network configuration resulting in certain customer services being interrupted and others maintained. The most significant change in the network configuration occurs when a network "bus" element fails and must be included in any reliability model.


CONSUMER ELEMENT
NETWORK ELEMENT
SOURCE ELEMENT
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
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30
31
32
33
34
35
36
37
38

Figure 2.3 Element block diagram of IEEE 14 bus network.

Each element number shown in Figure 2.3 is characterized by a set of unique statistical distributions for its failure and renewal processes. In many cases, the parameters for these statistical distributions can be empirically determined from a utility's outage reporting data base. In other cases, where the electrical equipment outage data is not available, or is not in readily usable form (e.g., utility's electrical equipment population is not know), average rates for the repair and failure processes, which are extracted from the literature, are used in the reliability analysis.

Once the element block diagram for a given network configuration has been drawn, the next step in analysis is to define the operational paths [17] to each individual customer element being serviced by the network configuration of Figure 2.2 as shown in Table 2.1 below. The most important conclusion that can be drawn from Table 2.1 is that each customer's service reliability level is unique because of the different number of operational paths servicing each customer location. Another important conclusion for utilities and their customers is : service reliability levels being delivered by a given network configuration are highly dependent upon a customer's location within the network and the number of operational paths linking the customer to the generation sources. For example, consumer element #13 has 11 operational paths while consumer element #10 has 64 operational paths [17].

Table 2.1 Number of operational paths to each consumer element in Figure 2.1

Consumer Element	Number of Operational Paths
2	22
3	20
4	38
6	34
7	34
8	54
9	54
10	64
11	54
12	54
13	11
14	26

These individual network operation paths to a customer element, for example element #13, are described by a PATH(i,j) array shown in Figure 2.4, where i is the path number and j are the elements forming the i-th path. This particular set of paths correspond to the normal operating network configuration. Each path traced in Figure 2.4 begins with a generation element and ends with the customer element selected (e.g., customer element #13).

j elements

i path	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	16	13																							
2	15	1	17	13																					
3	15	1	18	3	19	13																			
4	15	1	18	3	22	2	21	13																	
5	15	1	18	3	22	2	23	14	20	13															
6	15	1	18	3	24	25	26	4	32	9	33	8	39	7	34	35	5	36	2	21	13				
7	15	1	18	3	24	25	26	4	29	11	30	12	31	7	34	35	5	36	2	21	13				
8	15	1	18	3	24	25	26	4	27	10	28	11	30	12	31	7	34	35	5	36	2	21	13		
9	15	1	18	3	24	25	26	4	32	9	33	8	38	7	34	35	5	36	2	23	14	20	13		
10	15	1	18	3	24	25	26	4	29	11	30	12	31	7	34	35	5	36	2	23	14	20	13		
11	15	1	18	3	24	25	26	4	27	10	28	11	30	12	31	7	34	35	5	36	2	23	14	20	13

Figure 2.4 Network operational paths to customer #13 - normal network operating configuration.

Any loss of an element due to an outage or scheduled maintenance activities may or may not alter the network configuration significantly. For example, if generation element #16 is out of service due to maintenance, the number of operational paths to customer element #13 as shown in Figure 2.5 is reduced to 10. However, if element #35 (i.e., a transformer) is out of service due to maintenance, for example, the number of operational paths to customer #13 as shown in Figure 2.6 is significantly reduced to five operational paths.

j elements

i path	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	15	1	17	13																					
2	15	1	18	3	19	13																			
3	15	1	18	3	22	2	21	13																	
4	15	1	18	3	22	2	23	14	20	13															
5	15	1	18	3	24	25	26	4	32	9	33	8	39	7	34	35	5	36	2	21	13				
6	15	1	18	3	24	25	26	4	29	11	30	12	31	7	34	35	5	36	2	21	13				
7	15	1	18	3	24	25	26	4	27	10	28	11	30	12	31	7	34	35	5	36	2	21	13		
8	15	1	18	3	24	25	26	4	32	9	33	8	38	7	34	35	5	36	2	23	14	20	13		
9	15	1	18	3	24	25	26	4	29	11	30	12	31	7	34	35	5	36	2	23	14	20	13		
10	15	1	18	3	24	25	26	4	27	10	28	11	30	12	31	7	34	35	5	36	2	23	14	20	13

Figure 2.5 Network operational paths to customer #13 - source element-#16 out of service

j elements

i path	1	2	3	4	5	6	7	8	9	10
1	16	13								
2	15	1	17	13						
3	15	1	18	3	19	13				
4	15	1	18	3	22	2	21	13		
5	15	1	18	3	22	2	23	14	20	13

Figure 2.6 Network operational paths to customer #13 - transformer element #35 out of service

2.3.3 Service Reliability Calculation - Passive Failure Mode

Service reliability levels are higher than the overall network (i.e., system) level. The system level accounts for all electrical equipment outages while individual service reliability levels are only a sub-set of the system reliability level. The sub-set includes only those outages that directly result in the interruption of service to that individual service location and those outages which do not result in a discontinuity of service are excluded from the sub-set. Power supply disturbances may be generated during the isolation and restoration activities of those outages (i.e., initially excluded from the sub-set), which may affect the performance of computerized processes. These outages must be included in the sub-set of the service reliability levels.

The reliability criterion used in this thesis assumes that continuity of service to a single service entrance is maintained provided at least one operational path is energized. This criterion can be altered to include any set number of paths or identification of particular paths within the network configuration. Once the continuity of service criteria have been defined, it is necessary to validate these assumptions by conducting load flow studies to insure that the voltage levels fall within acceptable standards. Transient studies are undertaken to ensure that during the isolation and network reconfiguring process the power system disturbances (e.g., sags and surges) [20-21] generated do not significantly affect customer's "sensitive" equipment (i.e., fall within the Computer Business Equipment Manufacturer Association (CBEMA) susceptibility characteristics of computers) [73].

In order to demonstrate the significant impact of changes in the Weibull distribution parameters on service reliability levels, each element in the network configuration was assumed to have a characteristic life parameter as shown in Table 2.2 and Table 2.3.

Table 2.2 Component Reliability Indices [22]

Type of Component	Failure Rate (f/yr)	Repair/restoration Time (hrs)
Line	0.050	23.0
Transformer	0.012	168.0
Bus	0.007	3.5
Supply (i.e., source)	0.100	1.1

Table 2.3 Scale parameter associated with the element in Figure 2.3

Element Number	Scale parameter for failure rate (year)	Scale parameter for repair rate (hours)
1	142.7	3.5
2	142.7	3.5
3	142.7	3.5
4	142.7	3.5
5	142.7	3.5
6	142.7	3.5
7	142.7	3.5
8	142.7	3.5
9	142.7	3.5
10	142.7	3.5
11	142.7	3.5
12	142.7	3.5
13	142.7	3.5
14	142.7	3.5
15	10.0	1.1
16	10.0	1.1
17	20.0	23.0
18	20.0	23.0
19	20.0	23.0
20	20.0	23.0
21	20.0	23.0
22	20.0	23.0
23	20.0	23.0
24	20.0	23.0
25	83.3	168.0
26	20.0	23.0
27	20.0	23.0
28	20.0	23.0
29	20.0	23.0
30	83.3	168.0
31	20.0	23.0
32	20.0	23.0
33	20.0	23.0
34	20.0	23.0
35	20.0	23.0
36	20.0	23.0
37	20.0	23.0
38	20.0	23.0

Note that the scale parameters (α) are assumed to be the same for the type of components. Recall that $\alpha = 1/\lambda$ only when the shape parameter is equal to one.

2.3.3.1 Test Results

The service reliability levels were evaluated from the Weibull statistical properties presented in Section 2.2. The frequency of interruption and duration per interruption experienced by customer element #13 at various time periods are shown in Figures 2.7, 2.8 and 2.9, during the one year study period. During the one year period, scheduled maintenance activities were assumed to be performed on network elements during predefined time intervals as shown below. Transient outages that occurred during the one year period were also considered. The occurrence of transients are usually random and of short duration. Therefore, in order to simulate the impact of transient outages on the network configuration, the time of occurrence of transients and the components affected were randomly generated and the duration of these outages are based on utilities data and existing publications. The randomly generated transient times and components affected, and the duration of the outages are shown below. The data needed for the proposed algorithm is not fixed and may be changed accordingly by the user. Note that the study was done for the first year of the network operating time period for purposes of illustration. The operating time for the algorithm is not fixed (e.g., the study duration of 5 years and starting time from the tenth year onwards).

a) Scheduled Maintenance Time and Component :

1) April 26, 09: 00 hours to April 30, 16: 00 hours
   - Source # 15 under maintenance

2) May 17, 08: 30 hours to May 19, 18: 00 hours
   - Transformer # 25 under maintenance

3) June 22, 10: 00 hours to June 25, 19: 00 hours
   - Bus # 3, # 7, # 10, and Line # 24, # 38 under maintenance

4) August 23, 09: 00 hours to August 29, 12: 00 hours
   - Source # 16 under maintenance

5) October 4, 12: 00 hours to October 11, 09: 00 hours
   - Transformer # 35 under maintenance

6) December 1, 10: 00 hours to December 3, 13: 00 hours
   - Bus # 2, and Line # 19 under maintenance

b) Random Transient Outages (i.e., generated from a c.d.f. of transient outages):

1) February 16, 12: 37 hours for 20 minutes
   - Element # 4 affected

2) July 1, 21: 56 hours for 10 minutes
   - Element # 12 affected

3) July 27, 06:44 hours for 15 minutes
   - Element # 35 affected
   (note that this occurs while Element # 16 is under maintenance)

4) October 19, 18:02 hours for 5 minutes
   - Element # 25 affected

For this particular study all elements were assumed to be new and operable at time $t = 0$ (i.e., assuming 100% reliable initially). In practice, each element's failure rate would be dependent upon its in-service operating history which would significantly affect the element failure rate (e.g., the element failure rate increases significantly with operating time, for example, when the shape parameter is 2.0). A detailed study of this impact is shown in Appendix C.

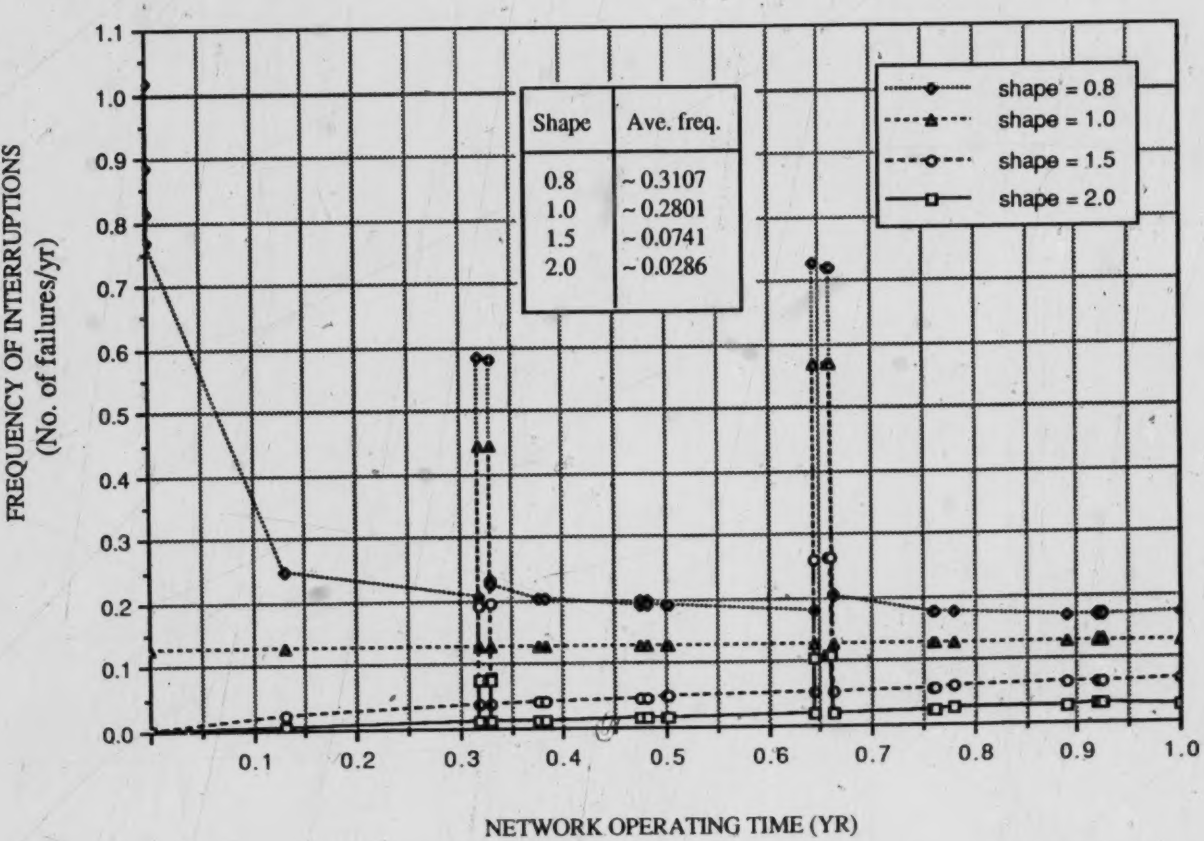
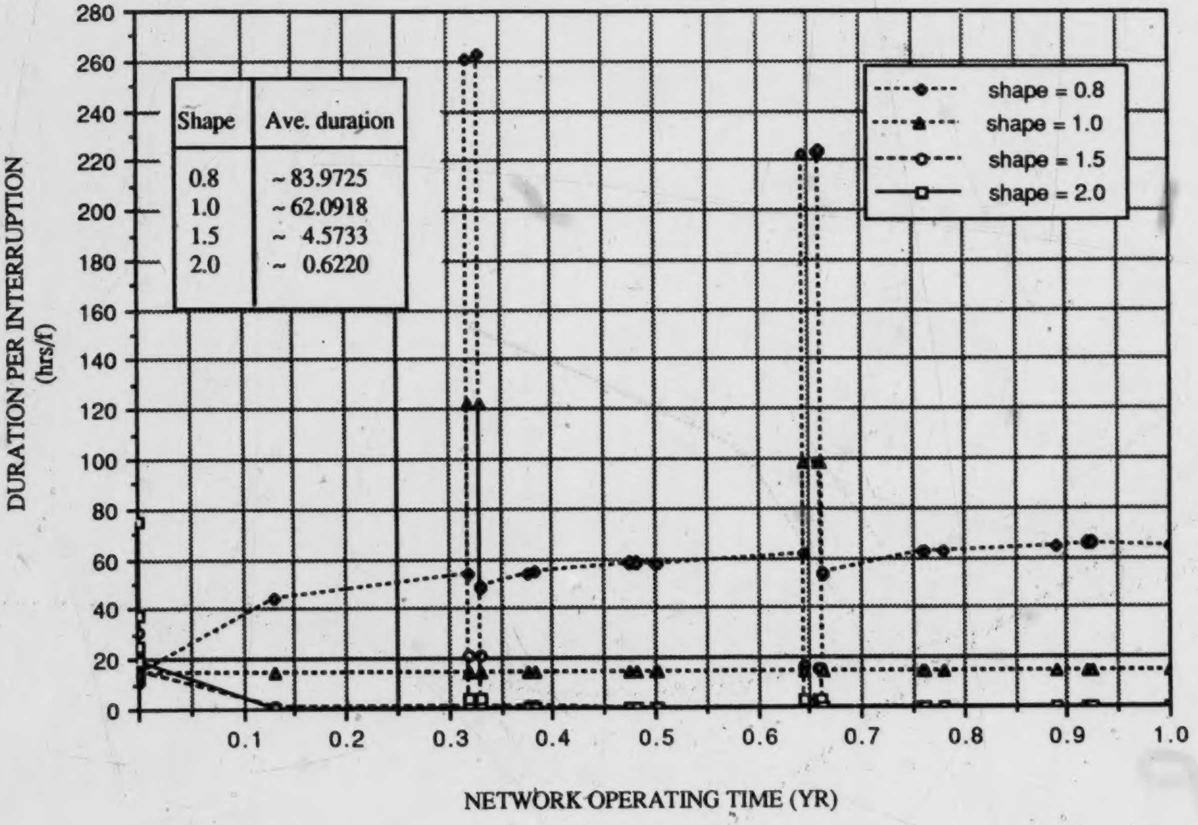


Figure 2.7 Frequency of interruptions experienced by customer element #13 at various operating time



Shape	Ave. duration
0.8	~ 83.9725
1.0	~ 62.0918
1.5	~ 4.5733
2.0	~ 0.6220

Figure 2.8 Duration per interruption experienced by customer element #13 at various operating time

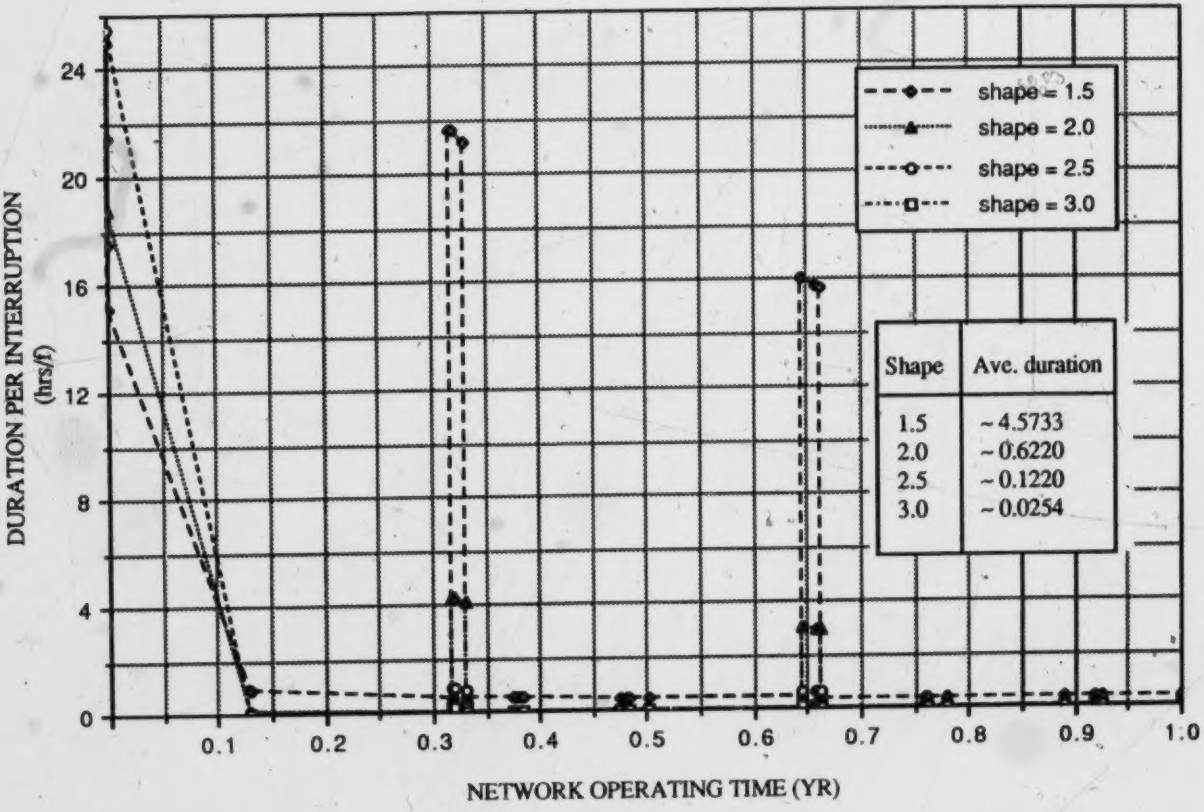


Figure 2.9 Duration per interruption experienced by customer element #13 at various operating time (shape parameter > 1.0)

2.3.3.2 Discussion of Results

It can be seen from Figure 2.7 that when the shape parameter $\beta = 1.0$ (i.e., exponential distribution), the frequency of customer element #13's interruptions is the long term average of ~ 0.2801 failure per year. However, when an element's lifetime Weibull distribution has a high variance (e.g., shape parameter of 2.0) characterized by an initial low failure rate, the frequency of customer element #13's interruptions is significantly lower (i.e., ~ 0.0286) during the first year of the network operation. The important point to note when the shape parameter is greater than 1.0 the frequency of customer interruptions gradually increases with an increase in the life of the network's components. When elements are removed for scheduled maintenance, the frequency of customer element #13's interruptions increases significantly (i.e., due to the new network operating configuration) which provides a significantly reduced number of operational paths to customer element #13 as shown in Figure 2.6. The level of impact from the scheduled maintenance depends on the element being maintained, i.e., the more crucial the element the greater the impact (see Figures 2.5 and 2.6).

The data of Figures 2.7 to 2.9 shows that the total annual duration of interruptions experienced by customer element #13 varies significantly with the changes in the shape parameter of the underlying statistical distribution representing an element's failure and repair processes. When the shape parameter equals one (i.e., exponential distribution representing an element's failure and renewal processes), the expected annual interruption duration experienced by customer element #13 is ~ 23.5 hours per year. In comparison, the total annual duration of interruption is significantly lower, ~ 0.128 hours per year, when the shape parameter of the Weibull distribution is 2.0. The lower values in the annual interruption duration (i.e., when the shape parameters are greater than 1.0) are due to the lower element failure rates during the initial operating years even though the average failure rate characterizing the statistical distribution are equal in all cases.

Notice the change in frequency of interruptions after the maintenance and transient outages on Figure 2.7 for the curves with Weibull distribution having shape parameter of 0.8, 1.5 and 2.0; however, with exponential distribution (i.e., Weibull distribution having $\beta = 1.0$) there is no change. In other words, if exponential distribution ($\beta = 1.0$) is used to characterize the failure and renewal processes of the components instead of Weibull statistical distribution, then the effect of preventive maintenance and transient outages during various period in the life of the distribution network can not be investigated. That is to say that if exponential distribution is used to characterize the failure and renewal processes, it does not matter whether the network has preventive maintenance or not, the effect still looks the same which is not true in practice. Therefore, it is better and more realistic to use a Weibull statistical distribution to characterize the failure, maintenance and renewal processes of the components in determining the customer level reliability indices, since the distribution allows determination of reliability indices anywhere in the time cycle of the network and allows variable rate parameters.

From Figure 2.7, the frequency of interruptions decreases in time when $\beta = 0.8$. This is because the components are in the debugging period of their life cycle (see Figure 2.1). When $\beta > 1.0$, the frequency of interruptions gradually increases with time. The greater the shape parameter values, the slower the rate of frequency of interruptions increases with time. As previously stated the shape parameter value depends on the component itself and the environment it is in. That is to say the shape parameter of the component may change as it ages. The operating time period of one year is used only for illustrating the Weibull distribution rate parameters as a function of time.

Consider the case where $\beta > 1.0$, i.e., component has been burned-in. The change (increase or decrease) in the frequency of interruptions after maintenance activities depends on the duration of maintenance and components being maintained (see Appendix C). If the stress exerted on the system due to the maintenance/transient outages is low, then preventive maintenance is useful i.e., regular preventive maintenance can improve the system performance. If, however, the stress exerted on the system due to the maintenance/transient outages is high, then preventive maintenance is not useful, since the activities cause the customer to experience a higher number of interruptions than is necessary.

To provide customers with an annual estimate of their service reliability levels, it is necessary to obtain a weighted average of the frequency and duration of their interruptions evaluated at the transition points in the reliability parameter (i.e., when maintenance and transient outages occur) versus operating time profile (e.g., Figure 2.7 to Figure 2.9).

2.3.4 Service Reliability Calculation - Including Active, Stuck Breakers and Common Mode Failures Events

Component outages can be divided into active and passive failures. All component faults which result in the removal of other "healthy" (i.e., working) components from service are classified as active failures. This class of failures includes component faults which cause operation of circuit breakers or disconnect switches. All component outages which do not remove any "healthy" components from service are classified as passive failures. These include undetected open failures (e.g., breakers stuck open), components out for repair, etc.. It should be noted that all first order active failures are classified as component passive failures. The stuck breaker failure mode corresponds to closed breakers failing to open successfully when requested to do so, i.e., breakers failing to respond because of a malfunction in the protection system, the relaying system or the breaker itself. If a breaker fails to open, other protection devices may respond, causing a breaker or breakers further from the faulted location to operate, which in turn may cause a greater proportion of the network configuration and additional load points to be disconnected. Common mode failure involves simultaneous outage of two or more components due to the same cause, i.e., an event having a single external cause with multiple failure effects which are not consequences of each other.

The impact of active, stuck breakers and common mode failure events in addition to the passive failure event were also examined in this thesis. The effect of weather conditions were also considered, e.g., whether to continue repair and maintenance activities during adverse weather period (e.g., repairing indoor equipment or outdoor equipment that are not sensitive to adverse weather), or to discontinue repair and maintenance activities in adverse weather periods (e.g., repairing outdoor equipment that is highly sensitive to adverse weather). The small system in Figure 2.10 is used to illustrate the impact of these failure modes on the customer reliability level when the failure and renewal processes of these failure modes parameters are of Weibull distribution. The rate parameters associated with the components in Figure 2.10 are shown in Table 2.4.

Table 2.4 Initial component rate parameters of Figure 2.10 [22]

Type of Component	Failure Rate (f/yr)		Repair Time (hrs/f)	
	Passive	Active	Passive	Active
Breaker	0.05790	0.01020	2.2	63.5
Disconnect	0.00165	0.00165	124.2	124.2
Transformer	0.01600	0.08060	611.0	15.5
Supply and node	assume 100 % reliable			

Normal weather duration = 200.0 hours
Adverse weather duration = 5.0 hours

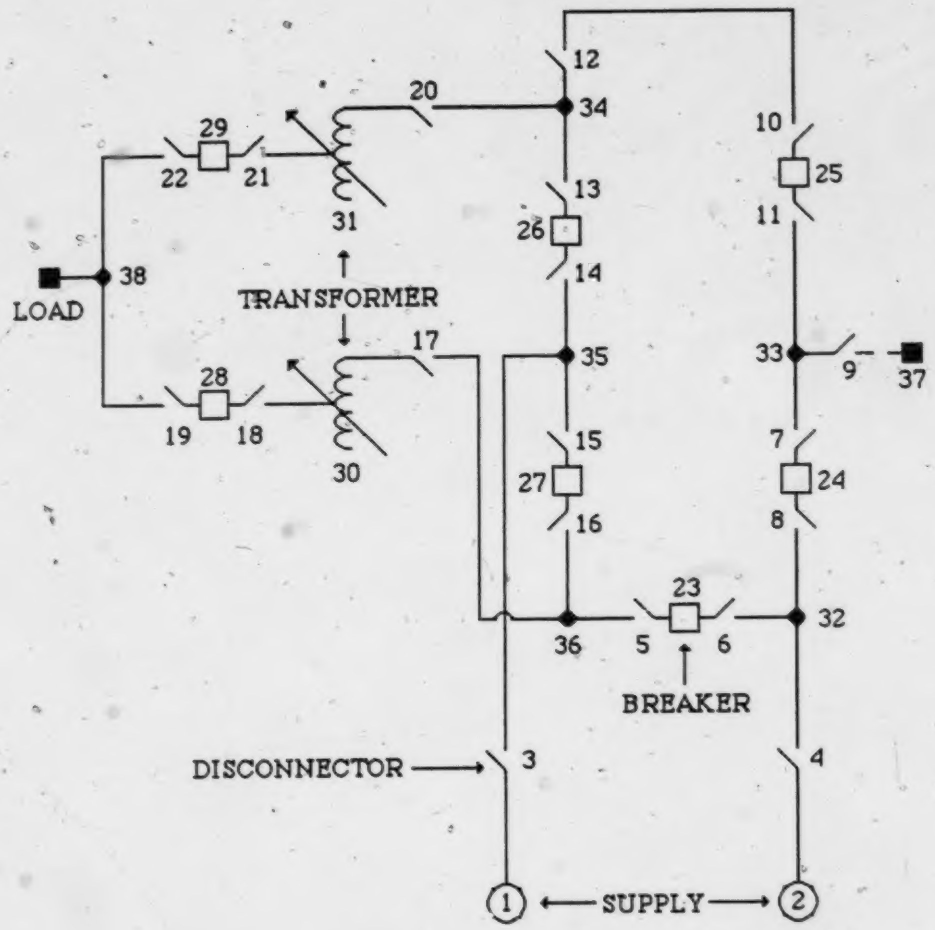

12
34
20
10
29
22
21
25
13
11
31
26
14
38
LOAD
TRANSFORMER
17
35
33
9
37
28
19
18
15
7
30
27
24
16
8
23
36
5
6
32
BREAKER
DISCONNECTOR
3
4
1
SUPPLY
2

Figure 2.10 Test system II.

2.3.4.1 Test Results

In this study, a maintenance activity on component # 30 is scheduled in the fourth month (1/3 year) for five hours, and a transient outage of five minutes was experienced by component # 31 at 0.889730 year. Note that the time of occurrence, durations and components affected by maintenance and transient activities are purely arbitrary and are used to illustrate the impact of overlapping failures due to passive, active, stuck breakers, and common mode events on the reliability indices for a given network configuration.

The frequency of interruptions experienced by customer element # 38, where the failure and renewal processes are characterized by the Weibull distribution having a shape parameter 2.0, over a one year operating period, are studied for the following three case studies :

CASE 1 : weather independent ;

CASE 2 : repair activities continued in adverse weather periods ;

CASE 3 : repair activities discontinued in adverse weather periods.

Note that shape parameter of 2.0 and operating time of one year are not of significance in these studies. They are just sample values used to demonstrate the use of Weibull statistical distribution in determining the consumer reliability indices of a power system under various modes of failure, i.e., determining the survival function of the utility supply to the consumers.

The impact of switching operation of breakers was also investigated in some detail. Switching operation of breakers is used for isolation of failed components, i.e., whether isolation of a failed component from the network configuration is possible or not possible. In practice, failed components are isolated either by using physically existing disconnects (isolators) or by disconnecting appropriate connections. In either case the protection breaker that has operated can be reclosed after the component is isolated. The frequency of interruptions for the above three cases and the possibility of switching operation (i.e., capability for isolating failed component), are shown in Tables 2.5, 2.6 and 2.7 respectively.

2.3.4.2 Discussion of Results

It can be seen from Tables 2.5 to 2.7 that the network configuration without switching operations (i.e., failed component can not be isolated) has a higher frequency of interruptions when compared to those network configurations where the isolation of failed components are possible. This is because, when switching is not possible, the network is interrupted for the repair or replacement time of the failed component; whereas when switching is possible, the network is only interrupted for the relevant isolation or switching time of the failed component(s).

In many practical studies the repair and maintenance activities may be aborted during adverse weather periods, for example, because of the severity of a storm and the possible risk in performing repair activities during these periods. In other cases, it may be possible to perform these activities during adverse weather periods, for example, the repair of indoor equipment failures. Comparing the results of the frequency of interruptions for customer element # 38 shown in Tables 2.5, 2.6 and 2.7, it is clear that the frequency of interruptions is slightly higher for Case 3 where repair activities are discontinued in adverse weather periods.

Whether repair and/ or maintenance activities are carried out during adverse weather conditions is an operational decision which must be established. The decision is dependent, for example, upon the nature of the individual customer and what impact the increased outage duration or frequency of interruptions will have on the customer processes. The decision is one of establishing an optimum policy in which reliability cost - reliability worth studies are required to secure a decision. It is clear from these results that selection of particular operating conditions from a reliability viewpoint is dependent upon how the successful operation is defined. This clearly illustrates the need to define very specifically the network operation criterion (e.g., predefined maintenance schedules) and the important indices of reliability for the judicious selection of a particular network configuration design (e.g., routing of distribution lines and load point locations).

Table 2.5 Frequency of interruptions experienced by customer element # 38 at various operating time period, with shape parameter of 2.0 (including passive, active, stuck breakers and common mode failure events) - weather independent

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.00011416	0.01000302	0.00000301
0.00022831	0.01000604	0.00000602
0.00034247	0.01000905	0.00000904
0.00045662	0.01001207	0.00001205
0.00057078	0.01001509	0.00001506
0.33287698	0.01879969	0.00878327
0.33299100	0.01880271	0.00878628
0.33310503	0.01880572	0.00878929
0.33321899	0.01880875	0.00879231
0.33333302	0.04943487	0.03942673
0.33344799	0.04944838	0.03944023
0.33356202	0.04946188	0.03945373
0.33367598	0.04947537	0.03946722
0.33379000	0.04948887	0.03948071
0.33390403	0.04950238	0.03949422
0.33401799	0.01855280	0.00853752
0.33413202	0.01855582	0.00854052
0.33424699	0.01855884	0.00854354
0.33436102	0.01856185	0.00854655
0.33447498	0.01856487	0.00854956
0.88972503	0.03324307	0.02320054
0.88972700	0.03324313	0.02320060
0.88972902	0.03324318	0.02320065
0.88973099	0.09900564	0.08899874
0.88973200	0.09900588	0.08899891
0.88973403	0.09900606	0.08899909
0.88973600	0.09900624	0.08899933
0.88973802	0.09900641	0.08899951
0.88973999	0.03324346	0.02320093
0.88974202	0.03324351	0.02320098
0.88974398	0.03324356	0.02320103
0.88974601	0.03324361	0.02320108
0.99965799	0.03614916	0.02610127
0.99977201	0.03615218	0.02610429
0.99988598	0.03615521	0.02610731
1.00000000	0.03615822	0.02611031
AVERAGE	~ 0.037343	~ 0.027287

Table 2.6 Frequency of interruptions per operating time periods experienced by customer element # 38 at various operating time period, with shape parameter of 2.0 (including passive, active, stuck breakers and common mode failure events) - repair activities continued in adverse weather

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.00011416	0.01219880	0.01219879
0.00022831	0.01220248	0.01220246
0.00034247	0.01220616	0.01220614
0.00045662	0.01220984	0.01220981
0.00057078	0.01221352	0.01221348
0.33287698	0.02293384	0.02291117
0.33299100	0.02293752	0.02291485
0.33310503	0.02294121	0.02291852
0.33321899	0.02294491	0.02292221
0.33333302	0.06032988	0.06031878
0.33344799	0.06034637	0.06033526
0.33356202	0.06036285	0.06035174
0.33367598	0.06037931	0.06036820
0.33379000	0.06039579	0.06038467
0.33390403	0.06041225	0.06040113
0.33401799	0.02263105	0.02260995
0.33413202	0.02263472	0.02261362
0.33424699	0.02263840	0.02261730
0.33436102	0.02264210	0.02262098
0.33447498	0.02264578	0.02262465
0.88972503	0.04057495	0.04051080
0.88972700	0.04057502	0.04051086
0.88972902	0.04057507	0.04051092
0.88973099	0.12084436	0.12083411
0.88973200	0.12084460	0.12083435
0.88973403	0.12084478	0.12083453
0.88973600	0.12084508	0.12083483
0.88973802	0.12084538	0.12083513
0.88973999	0.04057543	0.04051128
0.88974202	0.04057548	0.04051133
0.88974398	0.04057554	0.04051139
0.88974601	0.04057560	0.04051145
0.99965799	0.04412666	0.04405350
0.99977201	0.04413035	0.04405718
0.99988598	0.04413405	0.04406086
1.00000000	0.04413773	0.04406454
AVERAGE	~ 0.045531	~ 0.045495

Table 2.7 Frequency of interruptions per operating time periods experienced by customer element # 38 at various operating time period, with shape parameter of 2.0 (including passive, active, stuck breakers and common mode failure events) - repair activities discontinued in adverse weather

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.00011416	0.01219881	0.01219880
0.00022831	0.01220249	0.01220248
0.00034247	0.01220618	0.01220616
0.00045662	0.01220987	0.01220984
0.00057078	0.01221356	0.01221352
0.33287698	0.02295565	0.02293166
0.33299100	0.02295933	0.02293534
0.33310503	0.02296303	0.02293902
0.33321899	0.02296674	0.02294272
0.33333302	0.06033318	0.06032129
0.33344799	0.06034965	0.06033777
0.33356202	0.06036614	0.06035425
0.33367598	0.06038261	0.06037071
0.33379000	0.06039909	0.06038718
0.33390403	0.06041555	0.06040365
0.33401799	0.02265221	0.02262987
0.33413202	0.02265589	0.02263355
0.33424699	0.02265958	0.02263723
0.33436102	0.02266328	0.02264092
0.33447498	0.02266697	0.02264460
0.88972503	0.04063254	0.04057040
0.88972700	0.04063259	0.04057046
0.88972902	0.04063265	0.04057051
0.88973099	0.12085229	0.12084216
0.88973200	0.12085253	0.12084240
0.88973403	0.12085271	0.12084258
0.88973600	0.12085301	0.12084287
0.88973802	0.12085325	0.12084317
0.88973999	0.04063301	0.04057087
0.88974202	0.04063306	0.04057093
0.88974398	0.04063313	0.04057099
0.88974601	0.04063319	0.04057105
0.99965799	0.04419206	0.04412209
0.99977201	0.04419576	0.04412578
0.99988598	0.04419947	0.04412948
1.00000000	0.04420315	0.04413316
AVERAGE	~ 0.045561	~ 0.045526

2.4 Conclusions

This chapter has presented a reliability study of a network whose element failure and renewal processes are characterized by the Weibull statistical distribution. This distribution allows the rate parameters associated with the distribution to be variable as opposed to the classical models which assume a constant value. The chapter clearly demonstrates the significant difference in the frequency of service interruptions based on the Weibull distribution (i.e., for various shape parameters but the same characteristic life).

Reliability results based on the exponential distribution are usually expressed as a long term average value independent of the network's electrical equipment in-service life. When the Weibull distribution is used in the reliability model, the frequency of service interruptions is dependent upon the in-service life of the network's equipment. The results presented in this chapter are for a study period of one year for the operation of the assumed "new" network configuration. It is important to note that during this operating period, the frequency of customer interruptions gradually increased with the in-service life of the network.

With the use of the Weibull statistical distribution in characterizing the failure and renewal processes of the network, it is possible to examine the impact of scheduled maintenance on the system (the jump/drop in frequency of interruptions after maintenance/transient activities on Figure 2.7). If an exponential distribution is used instead of the Weibull distribution, the effect of scheduled preventive maintenance cannot be seen (Figure 2.7 with $\beta = 1.0$ no change in frequency of interruptions after maintenance activities). The study clearly revealed the impact of a scheduled maintenance activity on a customer's service reliability level. The removal of electrical components from service for maintenance purposes will alter the network's operating configuration which may or may not significantly change the number of operational paths servicing an individual customer location. Changes in the number of operational paths servicing a given customer location is directly correlated with service reliability levels experienced by a customer. In practice, it is important to determine the impact of maintenance activities on the various network operating configurations (e.g., load flow and transient studies).

The operating policies of performing or not performing repair and maintenance activities during adverse weather periods were considered in some detail in this research. It was found that the frequency of interruptions was higher but not significantly higher when the repair and maintenance activities were discontinued in adverse weather periods. The impact of transient outages on the network configuration was also investigated. The frequency of interruptions due to transient outages is dependent on the component affected, that is, if the component affected is critical then the impact is severe. On the other hand, if the component affected happens to reduce the risk of interruptions due to the affected component and its connecting components to a particular customer, then the frequency of interruptions to this particular customer will be lower, while the frequency of interruptions to the other customers may be much higher.

The primary advantage of the use of the Weibull distribution for power system reliability modeling studies is its ability to model the varying rate parameters of a utility's electrical equipment to provide more accurate estimates of service reliability levels during various periods in the life of a utility's electrical system. The flexibility of the Weibull distribution to describe the non-stationary failure and renewal processes of electrical equipment as opposed to the use of exponential distribution provides better estimates of the characteristics of a utility's network performance.

CHAPTER III UNBALANCED THREE-PHASE LOAD FLOW

3.1 Introduction

Distribution networks are characterized by their variable operating configurations which are always in a state of connecting, disconnecting and interconnecting new and old customer loads. The electrical characteristics (e.g., voltage levels, load flow patterns, fault levels,etc.) of these variable network operating configurations must be evaluated by an electrical utility to determine if the quality of service they deliver falls within acceptable standards (e.g., ANSI Utility Power Profile : ANSI C84.1-1977, Electrical Power Systems Equipment 60 Hz., Canadian Electrical Association's Standard : Preferred Voltage Levels for A.C. Systems 0 - 500 kV, and CSA Standard C235-1969). One of the problems encountered in power system operation is the generation of unbalanced voltages and currents in distribution and transmission lines with few or no transpositions, including unbalances arising in source and load connections. Distribution system loads are unbalanced (i.e., load connections) because the majority of the loads they serve are single phase.

Unbalanced network operation can have a significant impact on network performance. For example, the presence of negative-sequence currents due to unbalance at the generator terminals may overheat the rotor of the generator. In addition, because of unpredictable current distributions, protective relays can malfunction and active power loss in the transmission lines could be higher than expected because of parallel untransposed transmission lines. Hence, to investigate these unbalance effects in any detail, a three-phase load flow solution that allows representation of all possible unbalances that can exist in a power system network is essential for power system planning.

Load flow models are used to calculate the power flow within a network and the voltage levels at various locations within a specified power system configuration subject to the regulating capability of generators, capacitors, and tap changing of transformers under load as well as specified net interchange between individual operating configurations within a utility's system. This information is essential for the evaluation of the performance of a power system and for analyzing the effectiveness of alternative plans for system expansion to meet increased load demand. These analyses require the calculation of numerous load flow studies for both normal and emergency operating conditions.

The usual load flow solution for utility planning studies is based on a balanced three-phase network operated under balanced three-phase generation and load conditions, that is, a single positive-sequence network is involved. In its more general form, the sequence component frame has also been applied in an attempt to include unbalanced effects [23-25]. This approach is restrictive because of the difficulty of representing transformers, generators and induction motors in the sequence component reference. In the present work, development of the three-phase load flow algorithm is associated with the phase frame of reference entirely, as this permits a full and comprehensive treatment.

The choice of method for any load flow solution depends on many factors, the most important being convergence characteristics, storage requirements, reliability (i.e., how dependable) and computer time. In this respect, the Newton-Raphson method possesses good convergence and reliability properties, as well as great generality and flexibility, which makes it very convenient when representing various conditions and constraints in the system [25-26]. However, it requires excessive storage and computation effort unless efficient sparsity techniques are employed. This method is also very sensitive to good estimates of initial conditions of the network. Due to the limitations of the computer memory available during this research project (for analysis of a large distribution network, ~ 200 buses) at the University of Alberta, the Gauss-Seidel Iteration method was employed in developing the three-phase load flow algorithm. A detailed comparison of the solution techniques for load flow problems can be found in [27, pp.325-332].

Many load flow programs presently available are used only for evaluation of balanced systems (i.e., single phase equivalent of a balanced three-phase system) [28-30]. Those that do allow three-phase calculations assume the lines are of three wires (phase a, b and c) and the components are symmetrical [24-25,31]. Some programs neglect the mutual coupling between phases and the mutual coupling between lines [24].

The three-phase load flow algorithm developed in this thesis consists of the following features :

(i) loads are entered as constant power sinks (or constant impedance load) in a specified balanced or unbalanced state ;

(ii) lines in the network are taken in their natural unbalanced mutually coupled form;

(iii) transpositions of lines can be accounted for ;

(iv) provision for investigation of different methods of voltage control at generator buses ;

(v) facilities for online tap changing of transformers are available;

(vi) as all branch elements are represented in general two-port Pi admittance matrix form, it is possible to incorporate any method of neutral point earthing and connection of transformers ;

(vii) in addition to the three-phase wires, provision for a neutral and a ground wire or two ground wires were also considered, with the earth as return.

3.2 Three-Phase Models of Components

3.2.1 *Transmission / Distribution Lines*

Transmission line parameters are calculated from the lines' geometrical configurations. The calculated parameters are expressed as a series impedance and shunt admittance per unit length of line, assuming a uniform line. The effect of ground current and earth wires are included in the calculation of these parameters.

SERIES IMPEDANCE

A three-phase transmission or distribution line with a neutral and a ground wire is illustrated in Figure 3.1(a). The following equations can be written for phase "a" :

$$V_a - V'_a = I_a(R_a + j\omega L_a) + I_b(j\omega L_{ab}) + I_c(j\omega L_{ac}) + I_n(j\omega L_{an}) + I_g(j\omega L_{ag}) - I_e(j\omega L_{ae}) + V_e$$

$$V_e = I_e\,(R_e + j\omega L_e) - I_a\, j\omega L_{ea} - I_b\, j\omega L_{eb} - I_c\, j\omega L_{ec} - I_n\, j\omega L_{en} - I_g\, j\omega L_{eg}$$

and substituting

$$I_e = I_a + I_b + I_c + I_n + I_g$$

$$V_a - V'_a = I_a\,(R_a + j\omega L_a) + I_b(j\omega L_{ab}) + I_c(j\omega L_{ac}) + I_n(j\omega L_{an}) + I_g(j\omega L_{ag})$$
$$- j\omega L_{ae}(I_a + I_b + I_c + I_n + I_g) + V_e$$

where :

V_m , V'_m - bus voltage at phase m with respect to earth, m = a, b, c, n, g

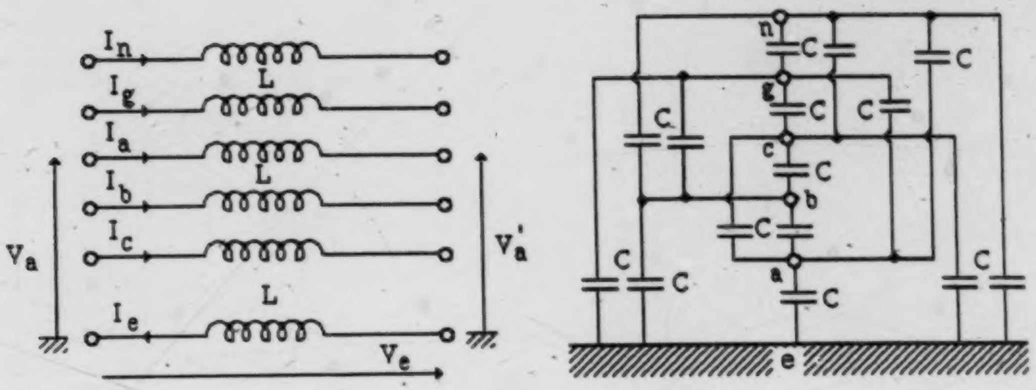
I_m - current flowing in phase m

R_m - phase m line self-resistance

L_m - phase m line self-inductance

L_{mp} - mutual impedance between phase m and p

ω = $2\pi f$, where f is the utility operating frequency



(a) (b)

Figure 3.1 Three-phase distribution line (a) series impedance equivalent ; (b) shunt admittance equivalent (assuming zero conductance, i.e., $g_c = 0$.)

Regrouping and substituting for V_e, that is,

$$\begin{aligned}\Delta V_a &= V_a - V'_a \\ &= I_a (R_a + j\omega L_a - j\omega L_{ae} + R_e + j\omega L_e - j\omega L_{ea}) \\ &\quad + I_b(j\omega L_{ab} - j\omega L_{ae} + R_e + j\omega L_e - j\omega L_{eb}) \\ &\quad + I_c(j\omega L_{ac} - j\omega L_{ae} + R_e + j\omega L_e - j\omega L_{ec}) \\ &\quad + I_n(j\omega L_{an} - j\omega L_{ae} + R_e + j\omega L_e - j\omega L_{en}) \\ &\quad + I_g(j\omega L_{ag} - j\omega L_{ae} + R_e + j\omega L_e - j\omega L_{eg})\end{aligned}$$

$$\begin{aligned}\Delta V_a = \; & I_a (R_a + j\omega L_a - 2j\omega L_{ae} + R_e + j\omega L_e) \\ & + I_b(j\omega L_{ab} - j\omega L_{be} - j\omega L_{ae} + R_e + j\omega L_e) \\ & + I_c(j\omega L_{ac} - j\omega L_{ce} - j\omega L_{ae} + R_e + j\omega L_e) \\ & + I_n(j\omega L_{an} - j\omega L_{ne} - j\omega L_{ae} + R_e + j\omega L_e) \\ & + I_g(j\omega L_{ag} - j\omega L_{ge} - j\omega L_{ae} + R_e + j\omega L_e)\end{aligned}$$

or

$$\Delta V_a = Z_{aa\text{-}e} I_a + Z_{ab\text{-}e} I_b + Z_{ac\text{-}e} I_c + Z_{an\text{-}e} I_n + Z_{ag\text{-}e} I_g \tag{3.1}$$

and writing similar equations for the other two phases, the following matrix equation results :

$$\begin{bmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \\ \Delta V_n \\ \Delta V_g \end{bmatrix} = \begin{bmatrix} Z_{aa\text{-}e} & Z_{ab\text{-}e} & Z_{ac\text{-}e} & Z_{an\text{-}e} & Z_{ag\text{-}e} \\ Z_{ba\text{-}e} & Z_{bb\text{-}e} & Z_{bc\text{-}e} & Z_{bn\text{-}e} & Z_{bg\text{-}e} \\ Z_{ca\text{-}e} & Z_{cb\text{-}e} & Z_{cc\text{-}e} & Z_{cn\text{-}e} & Z_{cg\text{-}e} \\ Z_{na\text{-}e} & Z_{nb\text{-}e} & Z_{nc\text{-}e} & Z_{nn\text{-}e} & Z_{ng\text{-}e} \\ Z_{ga\text{-}e} & Z_{gb\text{-}e} & Z_{gc\text{-}e} & Z_{gn\text{-}e} & Z_{gg\text{-}e} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \\ I_g \end{bmatrix} \tag{3.2}$$

Partitioning equation (3.2) as follows :

$$\left[\begin{array}{c} \Delta V_{abc} \\ \hline \Delta V_{ng} \end{array}\right] = \left[\begin{array}{c:c} Z_A & Z_B \\ \hdashline Z_C & Z_D \end{array}\right] \left[\begin{array}{c} I_{abc} \\ \hline I_{ng} \end{array}\right] \tag{3.3}$$

and assuming that the neutral and ground wires are at zero potential yeilds :

$$\Delta V_{abc} = Z_{abc} I_{abc} \tag{3.4}$$

where :

$$Z_{abc} = Z_A - Z_B Z_D^{-1} Z_C$$

SHUNT ADMITTANCE

With reference to Figure 3.1(b) the potentials of the line conductors are related to the conductor charges by matrix equation [26] :

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_n \\ V_g \end{bmatrix} = \begin{bmatrix} P_{aa} & P_{ab} & P_{ac} & P_{an} & P_{ag} \\ P_{ba} & P_{bb} & P_{bc} & P_{bn} & P_{bg} \\ P_{ca} & P_{cb} & P_{cc} & P_{cn} & P_{cg} \\ P_{na} & P_{nb} & P_{nc} & P_{nn} & P_{ng} \\ P_{ga} & P_{gb} & P_{gc} & P_{gn} & P_{gg} \end{bmatrix} \begin{bmatrix} Q_a \\ Q_b \\ Q_c \\ Q_n \\ Q_g \end{bmatrix} \tag{3.5}$$

Similar considerations as for the series impedance matrix lead to

$$V_{abc} = P'_{abc} Q_{abc} \tag{3.6}$$

where P'_{abc} is a 3 X 3 matrix which includes the effects of the ground wire. The capacitance matrix of the transmission line of Figure 3.2 is given by

$$C'_{abc} = \left[P'_{abc}\right]^{-1} = \begin{bmatrix} C_{aa} & -C_{ab} & -C_{ac} \\ -C_{ba} & C_{bb} & -C_{bc} \\ -C_{ca} & -C_{cb} & C_{cc} \end{bmatrix} \tag{3.7}$$

The series impedance and shunt admittance lumped-Π model representation of the three-phase line is shown in Figure 3.2, and its matrix equivalent is illustrated in Figure 3.3. These two matrices can be represented by compound admittances as shown in Figure 3.4.

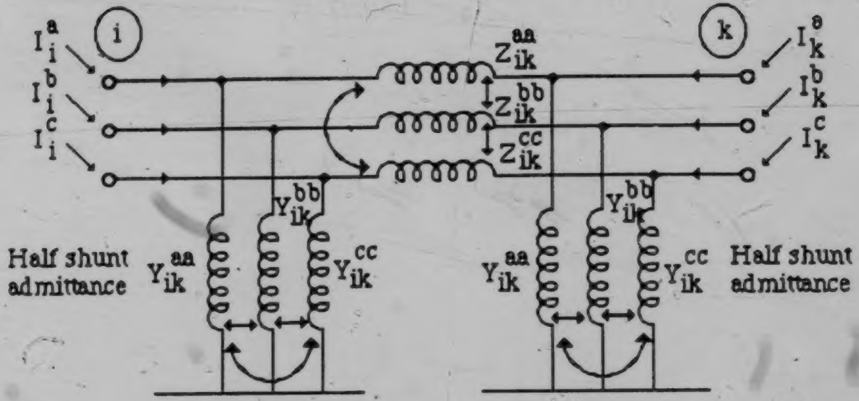
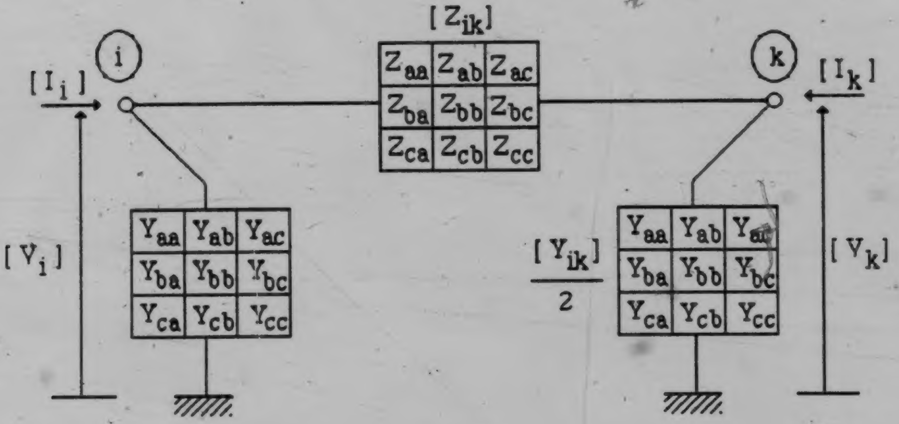

I_i^a
I_i^b
I_i^c
i
k
I_k^a
I_k^b
I_k^c
Z_{ik}^{aa}
Z_{ik}^{bb}
Z_{ik}^{cc}
Y_{ik}^{bb}
Half shunt admittance
Y_{ik}^{aa}
Y_{ik}^{cc}
Y_{ik}^{aa}
Y_{ik}^{bb}
Y_{ik}^{cc}
Half shunt admittance

Figure 3.2 Full circuit representation of the lumped-Π model of a short three-phase line


$[Z_{ik}]$
i
k
$[I_i]$
$[I_k]$
$[V_i]$
$[V_k]$
$\frac{[Y_{ik}]}{2}$

Z_{aa}	Z_{ab}	Z_{ac}
Z_{ba}	Z_{bb}	Z_{bc}
Z_{ca}	Z_{cb}	Z_{cc}

Y_{aa}	Y_{ab}	Y_{ac}
Y_{ba}	Y_{bb}	Y_{bc}
Y_{ca}	Y_{cb}	Y_{cc}

Y_{aa}	Y_{ab}	Y_{ac}
Y_{ba}	Y_{bb}	Y_{bc}
Y_{ca}	Y_{cb}	Y_{cc}

Figure 3.3 Matrix equivalent of Figure 3.2

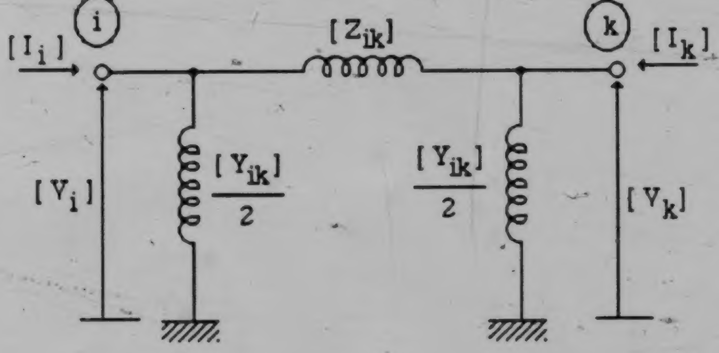

i
k
$[Z_{ik}]$
$[I_i]$
$[I_k]$
$[V_i]$
$\frac{[Y_{ik}]}{2}$
$\frac{[Y_{ik}]}{2}$
$[V_k]$

Figure 3.4 Using three-phase compound admittance of Figure 3.2.

Following the rules developed for the formation of the admittance matrix using the compound concept [26], the nodal injected currents of Figure 3.4 can be related to the nodal voltages by the equation,

$$\begin{bmatrix} [I_i] \\ \\ [I_k] \end{bmatrix} = \begin{bmatrix} [Z]^{-1} + \frac{[Y]}{2} & -[Z]^{-1} \\ \\ -[Z]^{-1} & [Z]^{-1} + \frac{[Y]}{2} \end{bmatrix} \begin{bmatrix} [V_i] \\ \\ [V_k] \end{bmatrix} \qquad (3.8)$$

This forms the element admittance matrix representation for the short line between busbars i and k in terms of 3 X 3 matrix quantities.

This representation may not be accurate enough for electrically long lines because of harmonics. The physical length at which a line is no longer electrically short depends on the wavelength of the distribution line. If harmonic frequencies are being considered, this physical length may be quite small. Using transmission line and wave propagation theory more exact models may be derived [32-36]. However, for a normal supply frequency analysis (i.e., 60 Hz), it is considered sufficient to model a long distribution/transmission line as a series of two or three nominal-Π sections [23].

3.2.2 *Three-Phase Models of Transformers*

The inherent assumption, that the transformer is a balanced three-phase device, is justified in the majority of practical situations, and traditionally, three-phase transformers are represented by their equivalent sequence networks.

Recently, methods have been developed [26,29,32-36] to enable all three-phase transformer connections to be accurately modelled in phase coordinates. In phase coordinates no assumptions are necessary although physically justifiable assumptions are still used in order to simplify the model. The primitive admittance matrix [26], used as a basis for the phase coordinate transformer model is derived from the primitive or unconnected network for the transformer windings and the method of linear transformation enables the admittance matrix of the actual connected network to be found.

Many three-phase transformers are wound on a common core and all windings are therefore coupled to all other windings. Therefore, in general, a basic two winding three-phase transformer has a primitive or unconnected network consisting of six coupled coils. If a tertiary winding is also present, the primitive network consists of nine coupled coils. The basic two-winding transformer shown in Figure 3.5 is now considered. The addition of more windings can be derived by extending the method above.

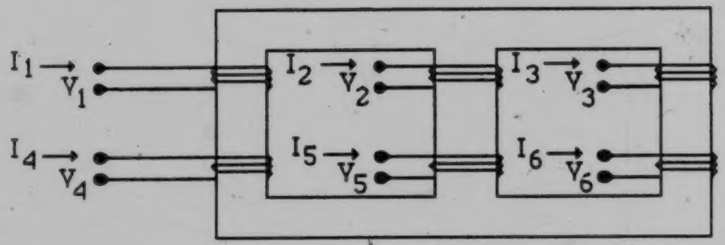


Figure 3.5 Diagrammatic representation of two-winding transformer.

The primitive network, Figure 3.6, can be represented by the primitive admittance matrix which has the following general form :

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} \\ y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} \\ y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \tag{3.9}$$

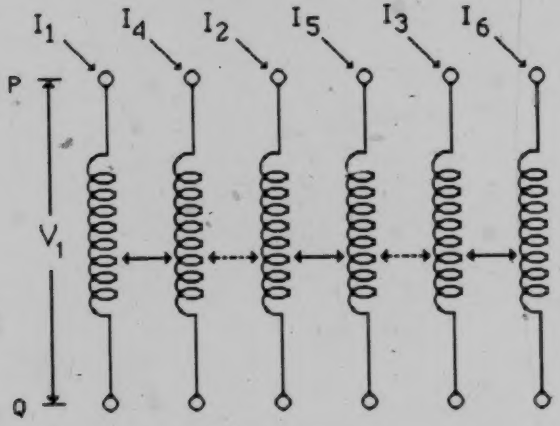


Figure 3.6 Primitive network of two-winding transformer. Six coupled coil primitive network. (Note the dotted coupling represent parasitic coupling between phases)

The elements of matrix [Y] can be measured directly, that is, by energizing coil *i* and short-circuiting all other coils, column *i* of [Y] can be calculated from $y_{ki} = I_k / V_i$.

By assuming that the flux paths are symmetrically distributed between all windings equation (3.9) may be simplified to equation (3.10).

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_p & y_m' & y_m' & -y_m & y_m'' & y_m'' \\ y_m' & y_p & y_m' & y_m'' & -y_m & y_m'' \\ y_m' & y_m' & y_p & y_m'' & y_m'' & -y_m \\ -y_m & y_m'' & y_m'' & y_s & y_m''' & y_m''' \\ y_m'' & -y_m & y_m'' & y_m''' & y_s & y_m''' \\ y_m'' & y_m'' & -y_m & y_m''' & y_m''' & y_s \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \tag{3.10}$$

where :

I_1 - the current flowing in line 1

V_1 - voltage across p and q

y_m' - is the mutual admittance between primary coils ;

y_m'' - is the mutual admittance between primary and secondary coils on different cores ;

y_m''' - is the mutual admittance between secondary coils.

In general, any two-winding three-phase transformer may be represented by two coupled compound coils. The network and admittance matrix for this representation is illustrated in Figure 3.7.

$$\begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} Y_{pp} & Y_{ps} \\ Y_{sp} & Y_{ss} \end{bmatrix} \begin{bmatrix} V_p \\ V_s \end{bmatrix} \tag{3.11}$$

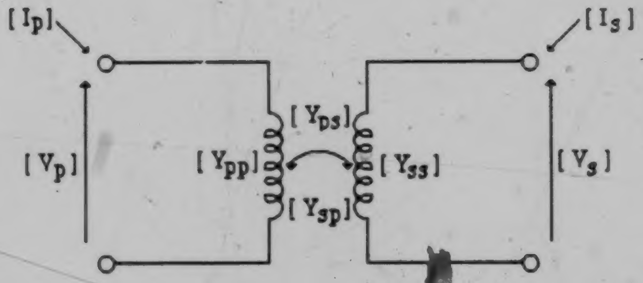


Figure 3.7 Two-winding three-phase transformer as two coupled compound coils

It should be noted that,

$$[\, Y_{sp} \,] = [\, Y_{ps} \,]^{T}$$

as the coupling between the two compound coils is equal and of opposite direction. Often, because more detailed information is not required or not available, the parameters of all three-phases are assumed balanced. In this case the common three-phase connections are found to be modelled by the basic submatrices. The submatrices, $[\, Y_{pp} \,]$, $[\, Y_{ps} \,]$ etc., are given in Table 3.1 for the common connections [26].

Table 3.1 Characteristic submatrices used in forming the transformer admittance matrices

Transformer connection		Self admittance		Mutual admittance
Bus P	Bus S	Y_{pp}	Y_{ss}	Y_{ps}, Y_{sp}
Wye - G	Wye - G	Y_I	Y_I	$-Y_{II}$
Wye - G	Wye	Y_I	$Y_{II}/3$	$-Y_{II}/3$
Wye - G	Delta	Y_I	Y_{II}	Y_{III}
Wye	Wye	$Y_{II}/3$	$Y_{II}/3$	$-Y_{II}/3$
Wye	Delta	$Y_{II}/3$	Y_{II}	Y_{III}
Delta	Delta	Y_{II}	Y_{II}	$-Y_{II}$

Basic submatrices used in node admittance formulation of common three-phase transformer connection, where :

$$Y_I = \begin{array}{|c|c|c|}\hline y_t & & \\ \hline & y_t & \\ \hline & & y_t \\ \hline \end{array} \qquad Y_{II} = \begin{array}{|c|c|c|}\hline 2y_t & -y_t & -y_t \\ \hline -y_t & 2y_t & -y_t \\ \hline -y_t & -y_t & 2y_t \\ \hline \end{array} \qquad Y_{III} = \begin{array}{|c|c|c|}\hline -y_t & y_t & \\ \hline & -y_t & y_t \\ \hline y_t & & -y_t \\ \hline \end{array}$$

Finally, these submatrices must be modified to account for off-nominal tap ratios as follows [26] :

(i) divide the self admittance of the primary by α^2

(ii) divide the self admittance of the secondary by β^2

(iii) divide the mutual admittance matrices by $(\alpha\beta)$.

where α and β are the primary and secondary tap ratio. It should be noted that in the per unit system a delta winding has an off-nominal tap of $\sqrt{3}$.

3.2.3 Parallel Lines Model

When two or more lines occupy the same right of way for a considerable length, the electrostatic and electromagnetic coupling between these lines must be considered in determining the load distribution. The electromagnetic coupling is represented by the coupled series elements while the capacitive coupling is represented the coupled shunt elements. These coupling parameters are lumped in a similar manner to the standard line parameters as shown in equation (3.8). The admittance matrix representation of parallel lines are :

$$\begin{bmatrix} [I_i] \\ [I_j] \\ [I_k] \\ [I_l] \end{bmatrix} = \begin{bmatrix} [Y_{11}]+[Y_{33}] & [Y_{12}]+[Y_{34}] & -[Y_{11}] & -[Y_{12}] \\ [Y_{12}]^T+[Y_{34}]^T & [Y_{22}]+[Y_{44}] & -[Y_{12}]^T & -[Y_{22}] \\ -[Y_{11}] & -[Y_{12}] & [Y_{11}]+[Y_{55}] & [Y_{12}]+[Y_{56}] \\ -[Y_{12}]^T & -[Y_{22}] & [Y_{12}]^T+[Y_{56}]^T & [Y_{22}]+[Y_{66}] \end{bmatrix} \begin{bmatrix} [V_i] \\ [V_j] \\ [V_k] \\ [V_l] \end{bmatrix}$$

where $[Y_{11}]$ is the m x m (m = number of phase) series admittance matrix quantity of line i-j ; and $[Y_{33}]$ and $[Y_{44}]$ the shunt admittance at bus i and j respectively. Similarly, $[Y_{22}]$ is the series admittance of line k-l ; and $[Y_{55}]$ and $[Y_{66}]$ the shunt admittance at bus k and l respectively. $[Y_{12}]$, $[Y_{34}]$ and $[Y_{56}]$ are the series and shunt mutual admittance between the parallel lines.

3.3 Power System Equations

The three-phase equations are an extension of the single phase equations, hence, only single phase equations will be discussed. The three-phase equations will be presented later in the chapter.

3.3.1 *Network Performance Equations*

The equation describing the performance of the network of a power system using the bus frame of reference in impedance form is :

$$E_{BUS} = Z_{BUS} I_{BUS} \tag{3.12}$$

or in admittance form is

$$I_{BUS} = Y_{BUS} E_{BUS} \tag{3.13}$$

The bus impedance and admittance matrices can be formed for the network including the ground wires (e.g., difference between neutral/ground wires and earth, etc.). The elements of the matrices, then, will include the effects of shunt elements to earth such as static capacitors and reactors, line charging, and shunt elements of transformer equivalents. When the earth is included and selected as the reference node, the bus voltages in the network performance equations (3.12) and (3.13) are measured with respect to earth.

If the earth is not included in the network, the elements of the bus impedance and admittance will not include the effects of the shunt elements and one of the buses of the network must be selected as the reference node. In this case, the effects of shunt elements are treated as current source at the buses of the network and the bus voltages in the performance equations (3.12) and (3..13) are measured with respect to the selected reference.

3.3.2 "s" Loading Equations

The real and reactive power at any bus p is defined as :

$$P_p - j\,Q_p = E_p^* I_p$$

and the current is :

$$I_p = \frac{P_p - jQ_p}{E_p^*} \qquad (3.14)$$

where I_p is positive when flowing into bus p.

In the formulation of the network equations, if the shunt elements to ground are included in the parameter matrix, then equation (3.14) gives the total current into the load bus. On the other hand, if the shunt elements are not included in the parameter matrix, the total current flowing into bus p is

$$I_p = \frac{P_p - jQ_p}{E_p^*} - y_p E_p \tag{3.15}$$

where y_p is the total shunt admittance at the bus and $y_p E_p$ is the shunt current flowing from bus p to ground.

3.3.3 *Line Flow Equations*

After the iterative solution of bus voltages is completed, line flows can be calculated. The current flowing in the line connecting bus p to q is

$$i_{pq} = (E_p - E_q)\, y_{pq} + E_p \frac{y'_{pq}}{2} \tag{3.16}$$

where y_{pq} = line admittance

y'_{pq} = total line charging admittance

$E_p\,(y'_{pq}/2)$ = current contribution at bus p due to line charging

The complex power flow (i.e., real and reactive) is given by the following equation :

$$S_{pq} = P_{pq} - j\,Q_{pq} = E_p^* i_{pq}$$

or

$$P_{pq} - j\,Q_{pq} = E_p^*(E_p - E_q)\, y_{pq} + E_p^* E_p \frac{y'_{pq}}{2} \tag{3.17}$$

where, at bus p, the real power flow from bus p to q is P_{pq} and the reactive power flowing is Q_{pq}. Similarly, at bus q, the complex power flow from q to p is :

$$P_{qp} - j\,Q_{qp} = E_q^*(E_q - E_p)\, y_{pq} + E_q^* E_q \frac{y'_{pq}}{2} \tag{3.18}$$

The power loss in line p-q is the algebraic sum of the power flows determined from equations (3.17) and (3.18).

3.4 Solution Technique

3.4.1 *Gauss-Seidel Iteration Using Y-Bus Matrix*

The solution of the load flow problem is initiated by assuming voltages (e.g., 1.0 p.u.) for all buses except the slack bus (i.e., where the voltage is specified and remains fixed). The currents flowing in the lines are calculated for all buses except the slack bus s from the bus loading equation [27] :

$$I_p = \frac{P_p - jQ_p}{E_p^*} \qquad p = 1, 2, 3, ..., n \quad \text{and} \quad p \neq s \tag{3.19}$$

where n is the number of buses in the network. The current flowing in the network can be obtained from the equation :

$$I_{BUS} = Y_{BUS}\, E_{BUS}$$

Selecting the ground as the reference bus, a set of n - 1 simultaneous equations can be written in the form :

$$E_p = \frac{1}{Y_{pp}} \left(I_p - \sum_{q=1,\ q \neq p}^{n} Y_{pq} E_p \right) \qquad p = 1, 2, 3, ..., n \quad \text{and} \quad p \neq s \tag{3.20}$$

The bus currents calculated from equation (3.19) above, the slack bus voltage, and the estimated bus voltages are substituted into equation (3.20) to obtain a new set of bus voltages. These new voltages are used in equation (3.19) to re-calculate bus currents for a subsequent solution of equation (3.20). The process is continued until changes between iterations in all bus voltages are negligible. After the voltage solution has been obtained, the power at the slack bus and line flows can be calculated. The line flows are calculated with the final bus voltages and the given line admittances and line charging from equations (3.17) and (3.18). The slack bus power can be determined by summing the flows on the lines terminating at the slack bus.

The network equation (3.20) and the bus loading equation (3.19) can be combined to obtain :

$$E_p = \frac{1}{Y_{pp}}\left(\frac{P_p - jQ_p}{E_p^*} - \sum_{q=1,\, q\neq p}^{n} Y_{pq} E_q\right) \qquad p = 1, 2, 3, \ldots, n \text{ and } p \neq s \tag{3.21}$$

in which only the bus voltages vary. Formulating the load flow problem in this manner results in a set of non-linear equations that can be solved iteratively.

A significant reduction in the computation time for a solution can be obtained by performing as many arithmetic operations as possible before initiating the iterative calculation. Letting

$$\frac{1}{Y_{pp}} = L_p$$

$$(P_p - j\,Q_p)\,L_p = KL_p$$

and

$$Y_{pq}\,L_p = YL_{pq}$$

then, the bus voltage equation (3.21) becomes :

$$E_p = \frac{KL_p}{E_p^*} - \sum_{q=1,\, q\neq p}^{n} YL_{pq}\,E_q \qquad p = 1, 2, \ldots, n \text{ and } p \neq s \tag{3.22}$$

Extending these equations to the three-phase system yields :

$$YL_{ij}^{pm} = \frac{Y_{ij}^{pm}}{Y_{ii}^{pp}} \tag{3.23}$$

$$KL_i^p = \frac{(S_i^p)^*}{Y_{ii}^{pp}} = \frac{P_i^p - jQ_i^p}{Y_{ii}^{pp}} \tag{3.24}$$

$$E_i^p = \frac{KL_i^p}{(E_i^p)^*} - \sum_{m=1,\ m\neq p}^{5} (YL_{ii}^{pm} E_i^m - \sum_{j=1,\ j\neq i}^{n} YL_{ij}^{pm} E_j^m) \tag{3.25}$$

where :

p, m - is phase a, b, c, neutral, ground (or 1, 2, 3, 4, 5)

i, j - is the bus number, 1 to n

n - is the total number of buses in the system

Y_{ij}^{pm} - is the element in the Y-Bus matrix corresponding to bus i-j and phase p-m

E_i^p - is the voltage at bus i, phase p

$(S_i^p)^*$ - is the complex conjugate power at bus i, phase p.

3.4.2 Acceleration of Convergence

In some cases the rate of convergence for an iterative process can be increased by applying an acceleration factor [27] to the approximate solution obtained from each iteration. Let "α" be the acceleration factor (usually ranging from 1.0 to 2.0) for the voltage, the new voltage equation for bus i, phase p becomes, for the k-th iteration :

$$[E_i^p]^{k+1} = [E_i^p]^k + \alpha \{ [E_i^p]^{k+1} - [E_i^p]^k \} \tag{3.26}$$

3.4.3 *Voltage Controlled Bus*

The voltage magnitude and the real power are specified at the voltage controlled buses. The reactive power at a voltage controlled bus p must be calculated before proceeding with the calculation of voltage at that bus. The single phase reactive bus power is [25] :

$$Q_p = e_p^2 B_{pp} + f_p^2 B_{pp} + \sum_{q=1,\, q\neq p}^{n} \{ f_p (e_p G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_p B_{pq}) \} \tag{3.27}$$

where e_p and f_p are the real and imaginary components of voltage at bus p, and must satisfy the relation :

$$e_p^2 + f_p^2 = [\, |E_p|_{(scheduled)} \,]^2 \tag{3.28}$$

and B_{pp} and G_{pp} are the real and imaginary components of the Y-Bus matrix at bus p.

The phase angle of the estimated bus voltage is :

$$\delta_p^k = \tan^{-1} \left(\frac{f_p^k}{e_p^k} \right) \tag{3.29}$$

Assuming that the estimated angles (eqn. 3.29) and the angles of scheduled voltages are equal, then adjusted estimates for e_p^k and f_p^k are :

$$e_p^k{}_{(new)} = |E_p|_{(scheduled)} \cos \delta_p^k \tag{3.30}$$

$$f_p^k{}_{(new)} = |E_p|_{(scheduled)} \sin \delta_p^k \tag{3.31}$$

Substituting the new E_p in equation (3.27), the reactive power Q_p^k is obtained and is used for calculating the new estimate E_p^{k+1}.

The limit of reactive power source at the voltage controlled bus is considered. If the calculated Q_p^k exceeds the maximum capability $Q_{p(max)}$ of the source the maximum value is taken as the reactive power at that bus. If the calculated value is less than minimum capability $Q_{p(min)}$ the minimum value is used.

The sequence of steps required for the load flow solution using the Gauss-Seidel Y-Bus matrix is shown in Figure 3.8.

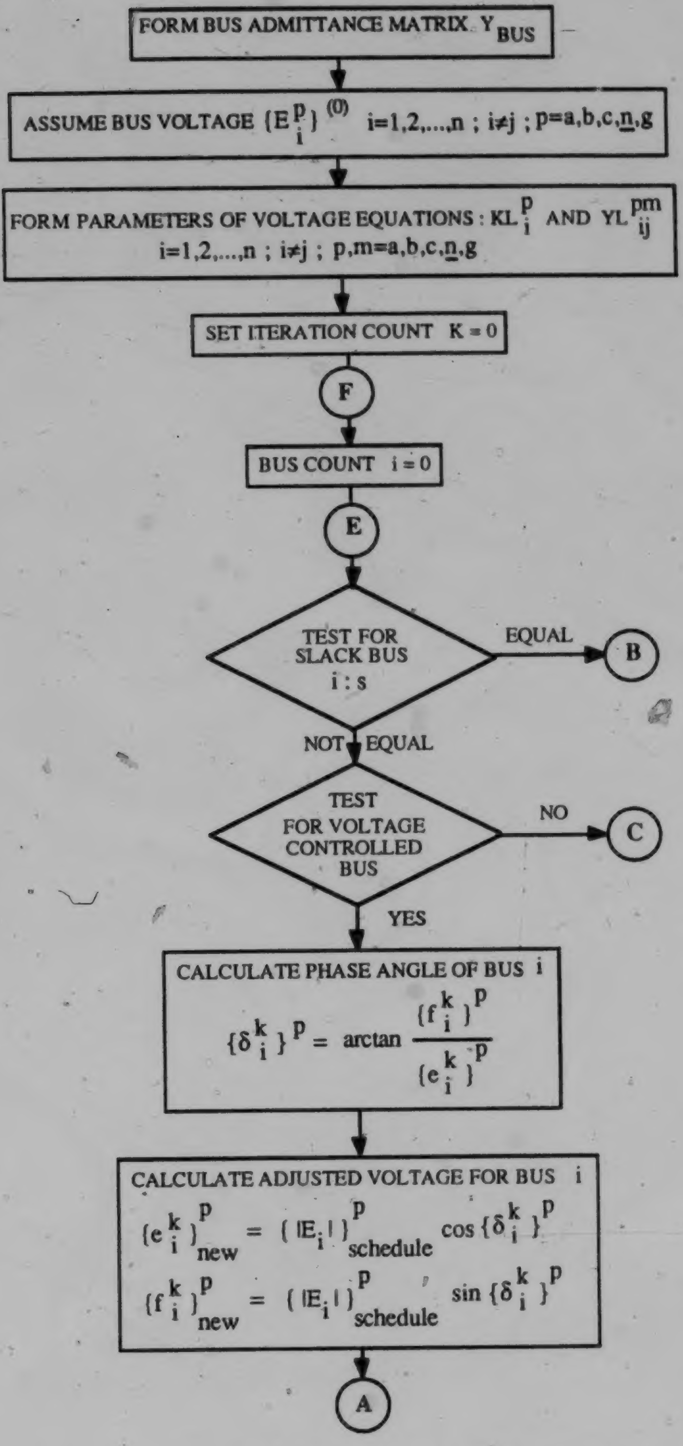

FORM BUS ADMITTANCE MATRIX Y_{BUS}
ASSUME BUS VOLTAGE $\{E_i^p\}^{(0)}$ i=1,2,...,n ; i≠j ; p=a,b,c,n,g
FORM PARAMETERS OF VOLTAGE EQUATIONS : KL_i^p AND YL_{ij}^{pm}
i=1,2,...,n ; i≠j ; p,m=a,b,c,n,g
SET ITERATION COUNT K = 0
F
BUS COUNT i = 0
E
TEST FOR SLACK BUS i : s
EQUAL
B
NOT EQUAL
TEST FOR VOLTAGE CONTROLLED BUS
NO
C
YES
CALCULATE PHASE ANGLE OF BUS i
$\{\delta_i^k\}^p = \arctan \frac{\{f_i^k\}^p}{\{e_i^k\}^p}$
CALCULATE ADJUSTED VOLTAGE FOR BUS i
$\{e_i^k\}^p_{new} = (|E_i|)^p_{schedule} \cos\{\delta_i^k\}^p$
$\{f_i^k\}^p_{new} = (|E_i|)^p_{schedule} \sin\{\delta_i^k\}^p$
A

Figure 3.8 Flow chart for the Gauss-Seidel iteration solution

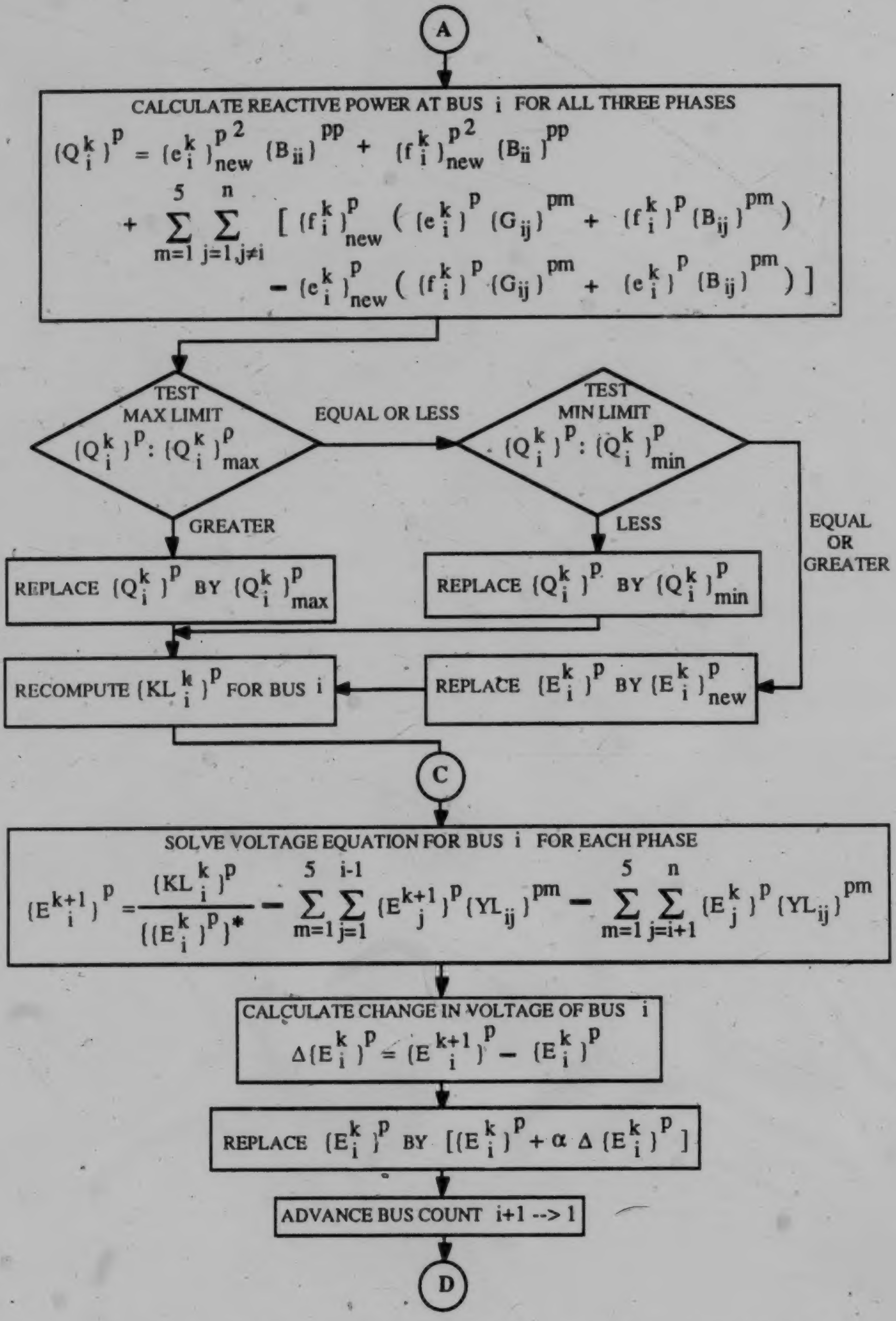

A
CALCULATE REACTIVE POWER AT BUS i FOR ALL THREE PHASES
$\{Q_i^k\}^p = \{e_i^k\}^{p\,2}_{new}\{B_{ii}\}^{pp} + \{f_i^k\}^{p\,2}_{new}\{B_{ii}\}^{pp} + \sum_{m=1}^{5}\sum_{j=1,j\neq i}^{n}\left[\{f_i^k\}^p_{new}\left(\{e_i^k\}^p\{G_{ij}\}^{pm} + \{f_i^k\}^p\{B_{ij}\}^{pm}\right) - \{e_i^k\}^p_{new}\left(\{f_i^k\}^p\{G_{ij}\}^{pm} + \{e_i^k\}^p\{B_{ij}\}^{pm}\right)\right]$
TEST MAX LIMIT
$\{Q_i^k\}^p : \{Q_i^k\}^p_{max}$
EQUAL OR LESS
TEST MIN LIMIT
$\{Q_i^k\}^p : \{Q_i^k\}^p_{min}$
GREATER
LESS
EQUAL OR GREATER
REPLACE $\{Q_i^k\}^p$ BY $\{Q_i^k\}^p_{max}$
REPLACE $\{Q_i^k\}^p$ BY $\{Q_i^k\}^p_{min}$
RECOMPUTE $\{KL_i^k\}^p$ FOR BUS i
REPLACE $\{E_i^k\}^p$ BY $\{E_i^k\}^p_{new}$
C
SOLVE VOLTAGE EQUATION FOR BUS i FOR EACH PHASE
$\{E_i^{k+1}\}^p = \frac{\{KL_i^k\}^p}{[\{E_i^k\}^p]^*} - \sum_{m=1}^{5}\sum_{j=1}^{i-1}\{E_j^{k+1}\}^p\{YL_{ij}\}^{pm} - \sum_{m=1}^{5}\sum_{j=i+1}^{n}\{E_j^k\}^p\{YL_{ij}\}^{pm}$
CALCULATE CHANGE IN VOLTAGE OF BUS i
$\Delta\{E_i^k\}^p = \{E_i^{k+1}\}^p - \{E_i^k\}^p$
REPLACE $\{E_i^k\}^p$ BY $[\{E_i^k\}^p + \alpha\,\Delta\{E_i^k\}^p]$
ADVANCE BUS COUNT i+1 --> 1
D

Figure 3.8 - continued : Flow chart for Gauss-Seidel iteration solution

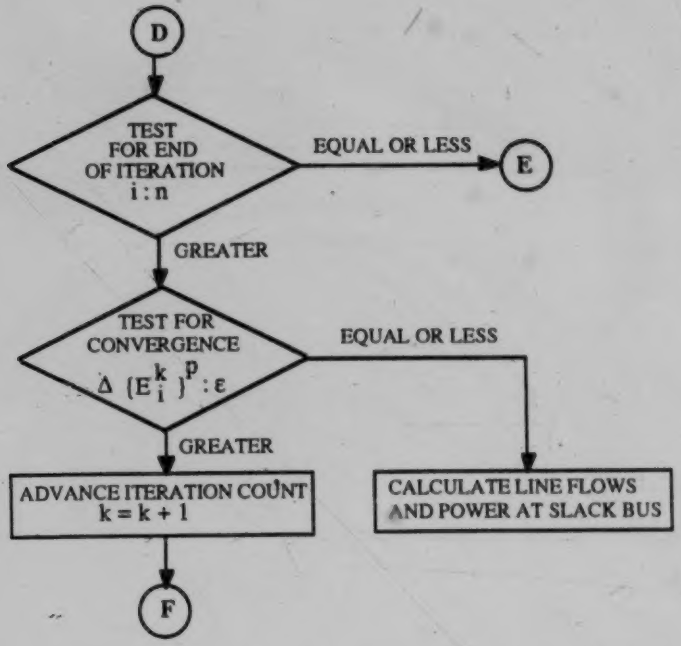

D
TEST FOR END OF ITERATION i : n
EQUAL OR LESS
E
GREATER
TEST FOR CONVERGENCE $\Delta \{E_i^k\}^p : \varepsilon$
EQUAL OR LESS
GREATER
ADVANCE ITERATION COUNT k = k + 1
CALCULATE LINE FLOWS AND POWER AT SLACK BUS
F

Figure 3.8 - continued : Flow chart for the Gauss-Seidel iteration solution.

3.5 Case Studies

The algorithm described was applied to a 14 bus test system shown in Figure 3.9. Sets of test data were abstracted from various publications [37-42] and transmission or distribution lines were assumed to be of a certain length (e.g., 1.0 km). Various sets of mutual coupling impedances / admittances, generation and load values were used to represent the unbalance of power flow in each phase of each bus within the test network configuration.

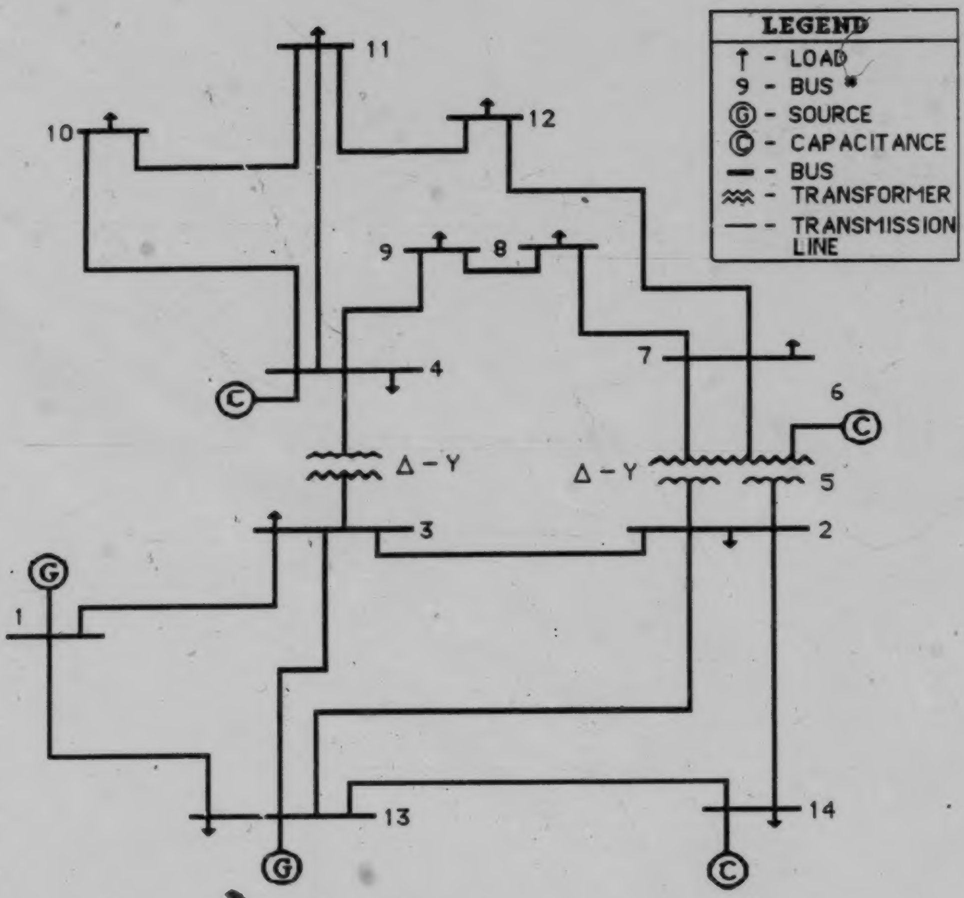

LEGEND
↑ - LOAD
9 - BUS
Ⓖ - SOURCE
Ⓒ - CAPACITANCE
▬ - BUS
TRANSFORMER
TRANSMISSION LINE
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Δ − Y
Δ − Y
G
G
C
C
C

Figure 3.9 Test system for unbalanced load flow studies.

3.5.1 System Definition

The test system used in the studies is assumed to have the configuration and operating conditions of Table 3.2.

Table 3.2 Test system configuration and operating conditions

(a) System Definition :

Number of buses = 14 Number of phases = 3

Number of transformers = 2 Number of control buses = 0

Number of transmission/distribution lines = 17

Number of wires per line = 5 (a, b, c, neutral, ground) *(b) Transformer Characteristics* :

- Delta-Wye connected
- $Z_{Xer} = 0.240 + j0.07$
- Tap settings are 1:1

(c) Evaluation criteria :

Tolerances = $0.00050 + j\ 0.00050$
Acceleration Factor = 1.40
$S_{base} = 100.0$ MVA
$V_{base} = 13.80$ kV
$V_{slack} = 1.06 + j0.0$ p.u.

Units of series impedances ($Z_{rs} = R_{rs} + j\ X_{rs,}$ r & s are phases) and shunt admittances ($Y_{rs} = j\ X_{rs}$) are in ohms and mhos, respectively.

Note that the transformer data and evaluation criteria (e.g., acceleration factor) are input data to the load flow program. An acceleration factor of 1.4 (an arbitrary value chosen for illustration purposes) is used for reducing the number of iterations for convergence. Note that the impact of the acceleration factor on number of iterations for convergence is non-linear, it varies according to the load flow solution method and network criteria. See reference [27] for the relationship between the number of iterations for convergence and acceleration factors.

A sample of the series impedances and shunt admittances for the transmission or distribution lines used in the studies is shown in Tables 3.3 and 3.4. Note that the tables shown are symmetrical, i.e., $Z_{ab} = Z_{ba}$; but $Z_{aa} \neq Z_{bb}$. The mutual coupling between phases are also included in the two tables. Table 3.5 shows the sample generation and load used.

Table 3.3 Series Impedance of transmission / distribution lines.

From Bus #	To Bus #	Phases					
		a	b	c	n	g	
1	3	0.121208 j0.498811	0.000000 j0.039885	0.000000 j0.039885	0.000000 j0.039885	0.000000 j0.039885	a
		0.000000 j0.039885	0.121045 j0.498811	0.000000 j0.039885	0.000000 j0.039885	0.000000 j0.039885	b
		0.000000 j0.039885	0.000000 j0.039885	0.121045 j0.498811	0.000000 j0.039885	0.000000 j0.039885	c
		0.000000 j0.039885	0.000000 j0.039885	0.000000 j0.039885	0.120556 j0.498811	0.000000 j0.039885	n
		0.000000 j0.039885	0.000000 j0.039885	0.000000 j0.039885	0.000000 j0.039885	0.121045 j0.498811	g
1	13	0.086419 j0.355642	0.000000 j0.028437	0.000000 j0.028437	0.000000 j0.028437	0.000000 j0.028437	a
		0.000000 j0.028437	0.086303 j0.355642	0.000000 j0.028437	0.000000 j0.028437	0.000000 j0.028437	b
		0.000000 j0.028437	0.000000 j0.028437	0.086303 j0.355642	0.000000 j0.028437	0.000000 j0.028437	c
		0.000000 j0.028437	0.000000 j0.028437	0.000000 j0.028437	0.085954 j0.355642	0.000000 j0.028437	n
		0.000000 j0.028437	0.000000 j0.028437	0.000000 j0.028437	0.000000 j0.028437	0.085954 j0.355642	g

Table 3.4 Shunt admittances of transmission / distribution lines.

From Bus	To Bus			Phases			
		a	b	c	n	g	
1	3	j0.004282	j0.000457	j0.000457	j0.000457	j0.000457	a
		j0.000457	j0.004283	j0.000457	j0.000457	j0.000457	b
		j0.000457	j0.000457	j0.004283	j0.000457	j0.000457	c
		j0.000457	j0.000457	j0.000457	j0.004283	j0.000457	n
		j0.000457	j0.000457	j0.000457	j0.000457	j0.004283	g
1	13	j0.006006	j0.000641	j0.000641	j0.000641	j0.000641	a
		j0.000641	j0.006007	j0.000641	j0.000641	j0.000641	b
		j0.000641	j0.000641	j0.006007	j0.000641	j0.000641	c
		j0.000641	j0.000641	j0.000641	j0.006007	j0.000641	n
		j0.000641	j0.000641	j0.000641	j0.000641	j0.006007	g

Table 3.5 Rated load / generation at each bus.

Bus #	Load (MVA)	Generation (MVA)
2	0.3	0.0
3	1.0	0.0
4	0.5	0.0
5	0.0	0.0
6	0.0	0.0
7	0.8	0.0
8	8.0	0.0
9	0.3	0.0
10	2.5	0.0
11	2.6	0.0
12	1.7	0.0
13	2.2	22.0
14	0.3	0.0

Note : - all loads are assumed to have power factor of 0.89 lag and all sources to have a power factor of 0.90 lag

- the loads at each bus are assumed to be balanced three-phase loads.

3.5.2 Impact of Mutual Impedances on Load Flow Solutions

3.5.2.1 Excluding Mutual Impedance Terms

The calculated values for the individual bus phase voltage are shown in Table 3.6 below. These values were calculated neglecting all the mutual impedance terms (i.e., a-b, b-c, a-n, etc.) in the model equations and assuming the loads at each bus were balanced.

Table 3.6 Phase voltages at each bus (mutual impedances neglected) - balanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.0	1.06000	240.00	1.06000	120.00
2	1.08814	-2.70	1.08816	237.30	1.08816	117.30
3	1.08496	-2.12	1.08497	237.88	1.08497	117.88
4	1.01384	-0.48	1.01384	239.52	1.01384	119.52
5	1.08830	-4.18	1.08833	235.82	1.08833	115.82
6	1.02783	-0.90	1.02783	239.10	1.02783	119.10
7	1.02614	-0.92	1.02614	239.08	1.02614	119.08
8	1.02589	-0.92	1.02590	239.08	1.02590	119.08
9	1.02285	-0.74	1.02285	239.26	1.02285	119.26
10	1.01216	-0.48	1.01216	239.52	1.01216	119.52
11	1.01238	-0.48	1.01239	239.52	1.01239	119.52
12	1.01826	-0.73	1.01826	239.27	1.01826	119.27
13	1.08699	-1.89	1.08701	238.11	1.08701	118.11
14	1.08674	-2.26	1.08676	237.74	1.08675	117.74

The power flowing (i.e., expressed in MVA) in each of the line sections in the IEEE 14 bus network is shown in Table 3.7 below. The phase current in each line is shown in Table 3.8 and the earth current is in Table 3.9.

Table 3.7 Power flowing in MVA in each phase of the network (mutual impedances neglected) - balanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	18.7756	-45.72	18.7763	-45.73	18.7758	-45.72
1	13	25.0628	-50.96	25.0635	-50.96	25.0632	-50.96
2	3	6.7342	158.43	6.7341	158.42	6.7343	158.42
2	5	23.9197	-15.59	23.9187	-15.59	23.9193	-15.59
2	13	10.4154	169.23	10.4160	169.23	10.4160	169.23
2	14	7.0751	164.42	7.0764	164.40	7.0759	164.40
3	1	18.5375	134.17	18.5382	134.16	18.5377	134.16
3	2	7.2862	-30.54	7.2862	-30.55	7.2865	-30.55
3	13	17.6029	204.12	17.6085	204.12	17.6093	204.13
4	9	27.8338	-78.08	27.8329	-78.08	27.8342	-78.09
4	10	1.2892	65.79	1.2890	65.80	1.2872	65.80
4	11	4.7029	43.20	4.6924	43.17	4.6941	43.03
5	2	23.4362	167.66	23.4355	167.66	23.4361	167.67
6	7	1.9126	-4.73	1.9124	-4.74	1.9108	-4.78
7	6	4.7813	246.54	4.7810	246.54	4.7794	246.55
7	8	12.0780	-83.50	12.0871	-83.50	12.0813	-83.46
7	12	6.2641	87.13	6.2643	87.12	6.2633	87.17
8	7	16.2534	265.18	16.2449	265.17	16.2522	265.15
8	9	11.6994	114.72	11.6994	114.70	11.6967	114.68
9	4	22.6062	104.58	22.6054	104.58	22.6065	104.57
9	8	21.2315	-76.58	21.2323	-76.59	21.2304	-76.60
10	4	5.9559	264.92	5.9557	264.92	5.9541	264.93
10	11	38.2689	-89.51	38.3286	-89.61	38.3543	-89.45
11	4	27.1164	262.75	27.1069	262.76	27.1009	262.74
11	10	11.8475	91.57	11.9077	91.25	11.9335	91.77
11	12	11.5971	-83.67	11.5958	-83.67	11.5950	-83.73
12	7	8.7232	268.12	8.7234	268.11	8.7225	268.14
12	11	8.0426	98.89	8.0414	98.90	8.0399	98.81
13	1	24.6645	129.07	24.6652	129.06	24.6649	129.06
13	2	10.7616	-17.60	10.7621	-17.60	10.7621	-17.60
13	3	16.1037	-2.92	16.1091	-2.91	16.1086	-2.91
13	14	10.0127	-47.09	10.0128	-47.08	10.0134	-47.08
14	2	7.8897	-30.06	7.8915	-30.07	7.8911	-30.07
14	13	7.3733	156.92	7.3735	156.92	7.3742	156.93

Note that the power per phase is equal in all cases (i.e., zero current in the neutral and ground conductor circuits).

Table 3.8 Current flowing in Amps. in phase a, b, and c (mutual impedances neglected) - balanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	1283.54	45.72	1283.59	-74.27	1283.55	165.72
1	13	1713.34	50.96	1713.34	-69.04	1713.37	170.96
2	3	448.46	198.87	448.44	78.88	448.45	-41.12
2	5	1592.90	12.90	1592.82	252.90	1592.86	132.89
2	13	693.60	188.07	693.63	68.07	693.63	-51.93
2	14	471.16	192.89	471.24	72.90	471.21	-47.10
3	1	1238.11	223.71	1238.14	103.72	1238.11	-16.28
3	2	486.64	28.42	486.64	268.43	486.65	148.43
3	13	1175.69	153.75	1176.05	33.76	1176.10	-86.25
4	9	1989.41	77.59	1989.35	-42.41	1989.44	197.60
4	10	92.15	-66.28	92.13	173.71	92.00	53.72
4	11	336.14	-43.68	335.39	196.35	335.51	76.49
5	2	1560.48	188.16	1560.39	68.16	1560.44	-51.84
6	7	134.84	3.83	134.83	243.84	134.71	123.87
7	6	337.65	112.54	337.62	-7.46	337.51	232.52
7	8	852.92	82.58	853.57	-37.42	853.16	202.54
7	12	442.36	-88.06	442.37	151.95	442.30	31.91
8	7	1148.06	93.90	1147.45	-26.09	1147.96	213.93
8	9	826.38	244.36	826.38	124.38	826.19	4.40
9	4	1601.54	254.68	1601.48	134.68	1601.55	14.69
9	8	1504.14	75.84	1504.20	-44.15	1504.07	195.86
10	4	426.40	94.60	426.39	-25.40	426.27	214.60
10	11	2739.79	89.03	2744.06	-30.87	2745.90	208.97
11	4	1940.92	96.77	1940.23	-23.24	1939.80	216.78
11	10	848.01	267.95	852.32	148.27	854.16	27.75
11	12	830.09	83.18	830.00	-36.82	829.93	203.25
12	7	620.78	91.15	620.79	-28.85	620.73	211.12
12	11	572.35	260.37	572.26	140.37	572.15	20.46
13	1	1644.24	229.04	1644.27	109.05	1644.25	-10.95
13	2	717.41	15.71	717.44	255.71	717.44	135.71
13	3	1073.54	1.03	1073.89	241.02	1073.86	121.01
13	14	667.49	45.20	667.49	-74.81	667.53	165.19
14	2	526.08	27.80	526.20	267.81	526.17	147.81
14	13	491.65	200.82	491.66	80.82	491.71	-39.19

An examination of the phase current levels shown in Table 3.8 reveals the phase currents are balanced when the mutual coupling terms and the neutral and ground conductors are neglected. The results of this case study will be compared with the results of the load flow model when these constraints are removed.

Table 3.9 Earth return current in Amp. flowing in the network (mutual impedances neglected) - balanced loads

BUS #		Mag.	Angle(°)
1	3	0.1271	-43.08
1	13	0.2113	-42.11
2	3	0.0738	247.92
2	5	0.1427	32.39
2	13	0.0411	266.39
2	14	0.1841	95.21
3	1	0.1156	217.60
3	2	0.0755	247.41
3	13	0.2376	-38.85
4	9	0.2894	260.64
4	10	0.1166	243.67
4	11	0.5140	223.53
5	2	0.1420	213.84
6	7	0.1686	264.80
7	6	0.1656	93.62
7	8	0.0806	-4.55
7	12	0.2963	264.61
8	7	0.0583	206.38
8	9	0.6030	143.41
9	4	0.2868	98.40
9	8	0.6148	-36.51
10	4	0.1167	118.08
10	11	1.7077	115.72
11	4	0.5104	40.54
11	10	1.6216	-66.85
11	12	0.8254	-60.26
12	7	0.2938	94.98
12	11	0.8228	119.67
13	1	0.2002	141.15
13	2	0.0430	264.23
13	3	0.2271	141.26
13	14	0.0687	237.72
14	2	0.1857	265.41
14	13	0.0740	236.21

Note that the earth currents are non-zero because of the minor mismatch at the final convergence, i.e., $|E_a| = |E_b| \pm \varepsilon$ as seen in Table 3.6 ; and also because of the difference in the phase impedances and admittances, i.e., $Z_{aa} \neq Z_{bb}$.

3.5.2.2 Including Mutual Impedance Terms

The phase voltage levels at each bus in the network, based on the mutual impedance terms and the neutral and ground conductors being included in the load flow model are shown in Table 3.10. If the voltages are unbalanced, then the line to line voltages at the three-phase loads will also be unbalanced. The loads on the IEEE 14 bus network configuration were balanced. In distribution feeder circuits and networks, the majority of the loads are single phase loads. The diversity in demand of the phase loads, however, creates unbalanced conditions in the network which can result in the phase voltages in various parts of the network being extremely unbalanced (e.g., 20%).

Table 3.10 Phase voltages at each bus (mutual impedance included) - balanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.00	1.06000	240.00	1.06000	120.00
2	1.09334	-2.64	1.09336	237.36	1.09336	117.36
3	1.08936	-2.09	1.08937	237.91	1.08937	117.91
4	1.02204	-0.71	1.02204	239.29	1.02205	119.29
5	1.09362	-4.00	1.09365	236.00	1.09365	116.00
6	1.03834	-1.20	1.03834	238.80	1.03834	118.80
7	1.03667	-1.21	1.03668	238.79	1.03668	118.79
8	1.03659	-1.21	1.03659	238.79	1.03660	118.79
9	1.03273	-0.99	1.03273	239.01	1.03273	119.01
10	1.01965	-0.67	1.01965	239.33	1.01966	119.33
11	1.01994	-0.68	1.01994	239.32	1.01995	119.32
12	1.02731	-0.98	1.02731	239.02	1.02732	119.02
13	1.09142	-1.88	1.09143	238.12	1.09143	118.12
14	1.09189	-2.25	1.09190	237.75	1.09191	117.75

Comparing these results with those of Table 3.6, it can be seen that the phase voltages are much higher, but not significantly unbalanced. Note that the level of unbalance depends on the size of mutual coupling impedances/admittances between the phases. In the study the mutual coupling impedances/admittances between the phases are small comparing to the size of the system, hence only a slight unbalance is experienced in the system.

Table 3.11 shows the power flowing in each of the line sections in the 14 bus test system. The currents through the neutral and ground conductors in this case are not zero but is of the magnitude shown in Table 3.12, and flow in the opposite direction to the currents in phase a, b and c (see Figure 3.10). The currents flowing in phase a, b and c are given in Table 3.13 and the earth return current at each bus is shown in Table 3.14.

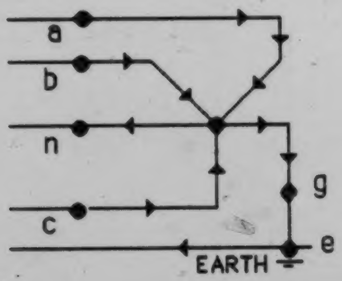


Figure 3.10 Direction of phase current flowing into a bus.

An examination of the results reveal that the earth current flowing in the network line sections is significantly higher when the mutual coupling and neutral and ground conductors are included in the load flow model. An unbalanced power flow can result in a significant current flowing in the neutral conductor, the ground conductor and the earth (Tables 3.12 and 3.14). If the neutral and ground currents are excessive, they may cause false tripping of the overcurrent ground fault relays at the substations resulting in disruption of the continuity of service to a utility's customers. This type of operating problem is difficult to detect in practice and can only be analyzed by an unbalanced load flow model that includes the mutual coupling terms, the neutral and the ground conductors.

Table 3.11 Power flowing in MVA in each phase of the network (mutual impedance included in model equations) - balanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	19.5999	-50.33	19.5984	-50.34	19.6037	-50.33
1	13	26.5630	-55.24	26.5613	-55.25	26.5688	-55.24
2	3	6.6348	153.83	6.6333	153.84	6.6357	153.83
2	5	22.3219	-16.13	22.3214	-16.13	22.3244	-16.13
2	13	9.9476	166.14	9.9461	166.15	9.9486	166.14
2	14	6.3067	163.84	6.3072	163.85	6.3085	163.84
3	1	19.4059	129.34	19.4044	129.33	19.4100	129.34
3	2	7.2778	-34.93	7.2761	-34.92	7.2785	-34.92
3	13	16.7741	206.98	16.7715	206.97	16.7752	206.98
4	9	32.3804	-79.41	32.3769	-79.41	32.3782	-79.42
4	10	2.8950	90.81	2.8922	90.84	2.8934	90.84
4	11	10.9853	85.90	10.9794	85.98	10.9747	85.93
5	2	21.8262	167.63	21.8256	167.63	21.8286	167.63
6	7	1.6105	-1.85	1.6082	-2.00	1.6084	-1.98
7	6	4.8592	250.68	4.8546	250.69	4.8553	250.69
7	8	13.5605	-89.00	13.5670	-89.00	13.5629	-88.98
7	12	7.7903	87.95	7.7893	87.95	7.7894	88.00
8	7	15.2267	269.11	15.2206	269.11	15.2251	269.09
8	9	15.8447	111.66	15.8416	111.66	15.8405	111.66
9	4	27.1062	102.50	27.1027	102.50	27.1039	102.50
9	8	25.6088	-76.67	25.6056	-76.67	25.6048	-76.67
10	4	7.7189	-89.67	7.7161	-89.66	7.7174	-89.66
10	11	46.0613	269.96	45.9498	269.87	46.0715	269.92
11	4	34.9906	268.73	34.9858	268.76	34.9809	268.74
11	10	19.2464	89.89	19.1366	89.68	19.2561	89.80
11	12	14.0315	-84.48	14.0298	-84.48	14.0276	-84.53
12	7	10.2747	268.66	10.2737	268.66	10.2740	268.70
12	11	10.4395	97.13	10.4378	97.13	10.4351	97.06
13	1	26.2453	124.52	26.2435	124.51	26.2514	124.51
13	2	10.3705	-21.00	10.3687	-21.00	10.3713	-21.00
13	3	14.9716	-1.84	14.9698	-1.85	14.9718	-1.83
13	14	10.3863	-51.83	10.3839	-51.84	10.3866	-51.84
14	2	7.1802	-32.31	7.1804	-32.30	7.1818	-32.31
14	13	7.3698	149.99	7.3674	150.00	7.3699	149.99

Table 3.12 Return current in the neutral and ground conductors in amperes (mutual impedance included in model equations) - balanced loads

BUS #		n		g	
I	J	Mag.	Angle(°)	Mag.	Angle(°)
1	3	0.0130	263.95	0.0130	263.95
1	13	0.0196	250.38	0.0196	250.38
2	3	0.0174	250.45	0.0174	250.45
2	5	0.0052	240.27	0.0052	240.27
2	13	0.0185	268.11	0.0185	268.11
2	14	0.0133	228.02	0.0133	228.02
3	1	0.0142	265.64	0.0142	265.64
3	2	0.0173	250.70	0.0173	250.70
3	13	0.0258	181.14	0.0258	181.14
4	9	0.0084	238.02	0.0084	238.02
4	10	0.0186	235.76	0.0186	235.76
4	11	0.0426	196.51	0.0426	196.51
5	2	0.0052	240.69	0.0052	240.69
6	7	0.0392	263.70	0.0392	263.70
7	6	0.0387	264.23	0.0387	264.23
7	8	0.0104	227.98	0.0104	227.98
7	12	0.0497	265.84	0.0497	265.84
8	7	0.0062	205.28	0.0062	205.28
8	9	0.0261	206.15	0.0261	206.15
9	4	0.0085	241.14	0.0085	241.14
9	8	0.0263	203.03	0.0263	203.03
10	4	0.0182	235.86	0.0182	235.86
10	11	0.8793	269.38	0.8793	269.38
11	4	0.0413	193.62	0.0413	193.62
11	10	0.8681	90.10	0.8681	90.10
11	12	0.0951	98.91	0.0951	98.91
12	7	0.0494	265.94	0.0494	265.94
12	11	0.0945	261.13	0.0945	261.13
13	1	0.0207	252.22	0.0207	252.22
13	2	0.0185	268.40	0.0185	268.40
13	3	0.0263	180.63	0.0263	180.63
13	14	0.0396	236.90	0.0396	236.90
14	2	0.0131	228.24	0.0131	228.24
14	13	0.0400	238.23	0.0400	238.23

Note that the neutral and ground conductors currents are equal because it was assumed that they are identical (see the line impedance/admittance characteristic in Tables 3.2 and 3.3). In general the neutral and ground conductors currents are different since the neutral and ground conductors impedance/admittance characteristic are different.

Table 3.13 Line Current at phase a, b and c in Amps. (mutual impedance included in model equations) - balanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	1339.88	50.33	1339.79	-69.66	1340.15	170.33
1	13	1815.90	55.24	1815.79	-64.75	1816.30	175.24
2	3	439.74	203.53	439.64	83.52	439.79	-36.47
2	5	1479.43	13.49	1479.38	253.49	1479.57	133.49
2	13	659.30	191.22	659.19	71.21	659.35	-48.78
2	14	417.99	193.52	418.02	73.51	418.10	-46.48
3	1	1290.88	228.57	1290.76	108.58	1291.13	-11.43
3	2	484.12	32.84	484.00	-87.17	484.16	152.83
3	13	1115.81	150.93	1115.62	30.94	1115.87	-89.07
4	9	2295.80	78.71	2295.55	-41.29	2295.64	198.71
4	10	205.26	268.48	205.06	148.45	205.15	28.45
4	11	778.87	-86.60	778.45	153.31	778.12	33.36
5	2	1446.21	188.36	1446.14	68.37	1446.33	-51.63
6	7	112.40	0.65	112.23	240.80	112.25	120.78
7	6	339.66	108.11	339.34	-11.90	339.39	228.10
7	8	947.88	87.79	948.33	-32.21	948.04	207.77
7	12	544.54	-89.16	544.47	150.83	544.48	30.79
8	7	1064.43	89.68	1064.00	-30.32	1064.31	209.70
8	9	1107.63	247.13	1107.42	127.13	1107.34	7.13
9	4	1901.98	256.51	1901.72	136.51	1901.80	16.52
9	8	1796.91	75.69	1796.68	-44.32	1796.61	195.69
10	4	548.56	89.00	548.36	-31.01	548.45	208.99
10	11	3273.45	89.37	3265.52	-30.55	3274.16	209.41
11	4	2485.98	90.59	2485.63	-29.44	2485.27	210.57
11	10	1367.40	269.43	1359.59	149.64	1368.08	29.52
11	12	996.89	83.80	996.77	-36.20	996.61	203.85
12	7	724.75	90.36	724.68	-29.65	724.69	210.32
12	11	736.37	261.89	736.25	141.89	736.06	21.96
13	1	1742.54	233.61	1742.40	113.62	1742.92	-6.39
13	2	688.54	19.13	688.41	259.12	688.58	139.12
13	3	994.03	-0.04	993.90	239.97	994.03	119.95
13	14	689.59	49.96	689.42	-70.04	689.60	169.96
14	2	476.52	30.06	476.52	-89.95	476.61	150.05
14	13	489.10	207.75	488.93	87.75	489.10	-32.24

Table 3.14 Earth return current in Amp. flowing in the network (mutual impedance included in model equations) - balanced loads

BUS #		Mag.	Angle(°)
1	3	0.1850	108.71
1	13	0.2784	122.30
2	3	0.2466	-61.19
2	5	0.0730	228.58
2	13	0.2626	258.86
2	14	0.1889	-35.33
3	1	0.2023	252.99
3	2	0.2460	118.56
3	13	0.3679	193.82
4	9	0.1199	109.28
4	10	0.2647	-43.08
4	11	0.6075	-3.84
5	2	0.0743	228.12
6	7	0.3838	-58.13
7	6	0.3784	121.34
7	8	0.1010	-22.46
7	12	0.5911	-63.12
8	7	0.0609	180.29
8	9	0.2552	231.79
9	4	0.1213	253.66
9	8	0.2576	48.64
10	4	0.2596	136.83
10	11	8.6373	116.21
11	4	0.5884	180.97
11	10	8.5267	-64.31
11	12	0.9308	-55.51
12	7	0.5876	116.79
12	11	0.9252	124.44
13	1	0.2942	-59.54
13	2	0.2629	100.93
13	3	0.3745	13.29
13	14	0.2188	95.80
14	2	0.1871	144.40
14	13	0.2204	265.52

Note that the load flow solutions (e.g., the line power and earth currents) on line 10-11 are significantly higher than the other lines. This is because the self and mutual impedances of line 10-11 used in the study are much lower than the other lines, while the self and mutual shunt admittances of line 10-11 are much higher than the other lines. This illustrates the impact the size of line impedance and admittance have on the load flow solutions.

3.5.3 Impact of Unbalanced Three-Phase Loads

The effect of unbalanced customer loads (e.g., single phase loads) on the distribution network power flow patterns were investigated using the load levels shown in Table 3.15. The individual bus phase voltages for the studies when the mutual impedance terms and the neutral and ground conductors are excluded and included are shown in Tables 3.16 and 3.17, respectively.

Table 3.15 Load and generator load levels at each bus (similar to Table 3.5, except that some of the loads are made single or double phase)

Generation (MVA)

Bus #	Phase a	Phase b	Phase c
13	19.363+j10.412	19.363+j10.412	19.363+j10.412

Load (MVA)

Bus #	Phase a	Phase b	Phase c
2	0.267+j0.137	0.267+j0.137	-
3	0.890+j0.456	-	-
4	-	-	0.445+j0.228
5	-	-	-
6	-	-	-
7	0.641+j0.365	-	-
8	-	6.230+j3.192	-
9	0.267+j0.137	0.267+j0.137	0.267+j0.137
10	-	2.225+j1.140	-
11	2.314+j1.185	2.314+j1.185	2.314+j1.185
12	1.530+j0.775	1.530+j0.775	1.530+j0.775
13	1.958+j1.003	1.958+j1.003	1.958+j1.003
14	0.267+j0.137	0.267+j0.137	0.267+j0.137

Table 3.16 Phase voltages in p.u. at each bus (mutual impedances neglected) - unbalanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.0	1.06000	240.00	1.06000	120.00
2	1.08914	-2.73	1.08986	237.31	1.09012	117.33
3	1.08692	-2.18	1.08767	237.87	1.08786	117.88
4	1.01802	-0.52	1.01779	239.39	1.01804	119.48
5	1.08929	-4.20	1.09008	235.84	1.09022	115.86
6	1.03430	-0.86	1.03119	239.02	1.03461	119.16
7	1.03265	-0.87	1.02961	238.99	1.03297	119.14
8	1.03254	-0.86	1.02926	238.99	1.03283	119.15
9	1.02856	-0.71	1.02616	239.17	1.02872	119.30
10	1.01531	-0.50	1.01418	239.46	1.01533	119.50
11	1.01656	-0.54	1.01536	239.42	1.01659	119.47
12	1.02370	-0.76	1.02144	239.17	1.02385	119.25
13	1.08822	-1.92	1.08976	238.09	1.08906	118.13
14	1.08770	-2.29	1.08925	237.72	1.08862	117.76

Table 3.17 Phase voltages in p.u. at each bus (mutual impedances included) - unbalanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.0	1.06000	240.00	1.06000	120.00
2	1.09324	-2.64	1.09410	237.40	1.09421	117.42
3	1.08928	-2.09	1.09015	237.95	1.09019	117.97
4	1.02354	-0.65	1.02229	239.31	1.02339	119.35
5	1.09345	-4.00	1.09441	236.03	1.09442	116.06
6	1.04168	-1.07	1.03876	238.82	1.04197	118.95
7	1.04004	-1.07	1.03710	238.81	1.04036	118.94
8	1.04014	-1.06	1.03698	238.81	1.04045	118.95
9	1.03533	-0.87	1.03304	239.03	1.03544	119.14
10	1.02100	-0.63	1.01988	239.34	1.02082	119.38
11	1.02130	-0.64	1.02018	239.33	1.02112	119.37
12	1.02950	-0.90	1.02763	239.03	1.02952	119.11
13	1.09135	-1.88	1.09211	238.16	1.09217	118.17
14	1.09181	-2.25	1.09261	237.78	1.09271	117.80

The line flow for these studies are given in Tables 3.18 and 3.19. Tables 3.20 and 3.21 show the current flow through each phase.

Table 3.18 Line flow in MVA in each phase of the network (mutual impedances neglected) - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	19.5486	-46.93	19.4671	-48.16	19.4397	-48.49
1	13	25.7529	-51.65	26.2318	-53.20	25.6725	-53.16
2	3	6.3637	163.23	6.4185	163.49	6.3515	162.96
2	5	23.8293	-15.60	23.9365	-15.72	23.8428	-15.47
2	13	10.3939	170.11	10.0563	173.40	10.3039	169.60
2	14	7.0319	164.14	6.6346	170.39	6.9480	163.67
3	1	19.3483	132.78	19.2639	131.55	19.2354	131.23
3	2	6.8351	-26.74	6.8845	-26.42	6.8294	-27.02
3	13	17.4715	194.71	17.2471	205.96	17.0198	194.41
4	9	30.5056	-86.01	25.8053	-79.90	30.7364	-86.86
4	10	3.5118	83.66	5.8761	99.11	3.5128	83.67
4	11	6.4422	23.76	14.7616	89.47	6.3866	23.68
5	2	23.3453	167.69	23.4490	167.55	23.3611	167.83
6	7	1.7311	-4.74	1.9792	-11.73	1.7416	-5.48
7	6	4.7801	248.87	4.5994	245.11	4.7641	248.69
7	8	12.4970	265.10	11.0964	-81.61	12.2205	267.12
7	12	6.8692	75.11	6.4760	85.26	7.0007	73.87
8	7	16.1432	-86.21	17.4817	264.69	16.3827	-87.85
8	9	12.4840	94.97	11.4379	112.45	12.7144	92.89
9	4	25.1560	94.68	20.4658	102.63	25.3858	93.63
9	8	22.6031	-87.15	21.1268	-77.97	22.8491	-88.29
10	4	8.2807	267.34	10.6038	-84.93	8.2819	267.35
10	11	152.551	269.02	148.699	87.64	152.746	268.83
11	4	27.1184	257.45	38.5616	269.82	27.0802	257.54
11	10	126.107	88.77	122.325	92.83	126.303	88.54
11	12	12.7035	269.27	11.8051	-84.85	12.7939	268.39
12	7	9.2853	259.24	8.9439	266.75	9.4043	258.26
12	11	9.1035	88.70	8.2209	97.15	9.1970	87.49
13	1	25.3892	128.26	25.8939	126.62	25.3043	126.75
13	2	10.7221	-16.79	10.3162	-13.92	10.6449	-17.36
13	3	17.2961	-12.01	15.5313	-1.88	16.9374	-13.02
13	14	9.9974	-45.23	9.9475	-45.32	9.9711	-45.79
14	2	7.8586	-30.41	7.2820	-25.85	7.7940	-31.00
14	13	7.4890	159.31	7.4329	159.46	7.4220	158.76

Table 3.19 Line flow in MVA in each phase of the network (mutual impedances included) - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	19.5765	-50.27	19.5719	-51.57	19.4953	-51.81
1	13	26.5259	-55.20	26.5736	-56.35	26.4841	-56.64
2	3	6.6028	153.95	6.6795	154.20	6.5983	153.56
2	5	22.2538	-15.99	22.3439	-16.20	22.2793	-15.98
2	13	9.9219	166.29	9.9431	165.93	9.8368	165.67
2	14	6.2903	164.09	6.2999	163.57	6.2031	163.46
3	1	19.3813	129.40	19.3780	128.10	19.2973	127.87
3	2	7.244	-34.86	7.3161	-34.54	7.2478	-35.23
3	13	16.8042	206.94	16.3035	206.85	16.3580	207.05
4	9	33.9974	-85.84	32.5127	-79.61	34.5801	-86.37
4	10	3.1397	82.17	2.9277	89.82	3.2153	83.05
4	11	12.3025	75.83	11.1368	84.82	12.6558	77.18
5	2	21.7617	167.78	21.8458	167.56	21.7868	167.80
6	7	1.4427	-0.25	1.6276	-2.91	1.4366	-2.82
7	6	4.8790	252.83	4.8393	250.40	4.8182	252.70
7	8	14.6321	261.50	13.3713	-86.88	14.6397	263.05
7	12	8.4840	79.22	7.8681	87.45	8.7415	78.55
8	7	14.6644	-81.52	15.4716	267.31	14.5694	-83.01
8	9	16.9823	98.18	16.0243	111.02	17.5393	96.52
9	4	28.6529	94.76	27.2342	102.26	29.2466	94.11
9	8	27.1644	-84.76	25.8172	-77.01	27.7422	-85.75
10	4	7.9582	266.94	7.7539	269.95	8.0354	267.25
10	11	46.7267	268.47	46.2466	269.52	46.6329	268.39
11	4	36.1406	265.24	35.1414	268.38	36.5255	265.61
11	10	19.8636	86.38	19.4234	88.83	19.7816	86.19
11	12	14.5369	-89.79	14.1063	-84.85	14.7475	269.52
12	7	10.9321	261.89	10.3521	268.28	11.1802	261.32
12	11	10.9238	89.97	10.5109	96.62	11.1396	89.05
13	1	26.2060	124.56	26.2576	123.39	26.1631	123.11
13	2	10.3417	-20.88	10.3712	-21.22	10.2721	-21.56
13	3	15.0034	-1.82	14.5787	-2.91	14.5974	-2.61
13	14	10.3634	-51.80	10.3935	-51.98	10.3498	-52.36
14	2	7.1567	-32.14	7.1833	-32.57	7.0944	-32.89
14	13	7.3509	150.10	7.3640	149.83	7.2972	149.46

Table 3.20 Line current in Amps. at phase a, b and c (mutual impedances neglected) - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	1336.38	46.93	1330.81	-71.84	1328.94	168.49
1	13	1760.52	51.65	1793.26	-66.80	1755.03	173.16
2	3	423.40	194.04	426.76	73.82	422.20	-45.63
2	5	1585.44	12.87	1591.52	253.03	1584.90	132.80
2	13	691.54	187.16	668.64	63.91	684.93	-52.27
2	14	467.85	193.13	441.13	66.93	461.86	-46.34
3	1	1289.93	225.04	1283.41	106.31	1281.30	-13.35
3	2	455.69	24.56	458.66	264.29	454.91	144.90
3	13	1164.80	163.12	1149.05	31.90	1133.71	-76.53
4	9	2171.43	85.48	1837.26	-40.71	2187.82	206.34
4	10	249.98	-84.18	418.36	140.27	250.04	35.81
4	11	458.57	-24.28	1050.98	149.91	454.60	95.80
5	2	1553.01	188.11	1558.79	68.28	1552.74	-51.97
6	7	121.28	3.88	139.08	250.75	121.98	124.64
7	6	335.43	110.26	323.70	-6.12	334.20	230.46
7	8	876.95	94.03	780.96	-39.39	857.28	212.03
7	12	482.03	-75.98	455.78	153.74	491.11	45.28
8	7	1132.93	85.35	1230.77	-25.70	1149.42	207.00
8	9	876.13	264.17	805.27	126.55	892.05	26.26
9	4	1772.28	264.60	1445.22	136.54	1788.19	25.66
9	8	1592.42	86.44	1491.90	-42.86	1609.50	207.58
10	4	591.00	92.16	757.65	-35.61	591.08	212.16
10	11	10887.78	90.48	10624.65	-32.90	10901.45	210.68
11	4	1933.08	102.01	2752.05	-30.40	1930.31	221.93
11	10	8989.28	-89.31	8730.04	146.59	9003.05	30.93
11	12	905.55	90.20	842.50	-35.73	911.96	211.07
12	7	657.27	100.01	634.51	-27.58	665.60	220.99
12	11	644.40	-89.46	583.21	142.02	650.92	31.76
13	1	1690.65	229.82	1721.82	111.46	1683.69	-8.62
13	2	713.98	14.87	685.97	252.00	708.29	135.49
13	3	1151.73	10.09	1032.75	239.97	1126.98	131.15
13	14	665.72	43.31	661.46	-76.59	663.46	163.92
14	2	523.55	28.12	484.44	263.58	518.80	148.76
14	13	498.93	198.40	494.48	78.26	494.05	-41.01

Table 3.21 Line current in Amps. at phase a, b and c (mutual impedances included) - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	1338.29	50.27	1337.98	-68.43	1332.76	171.81
1	13	1813.36	55.20	1816.63	-63.65	1810.51	176.64
2	3	437.66	203.41	442.40	83.19	436.97	-36.14
2	5	1475.07	13.36	1479.87	253.59	1475.44	133.40
2	13	657.66	191.07	658.55	71.47	651.44	-48.25
2	14	416.95	193.28	417.25	73.82	410.80	-46.04
3	1	1289.33	228.51	1288.08	109.85	1282.67	-9.91
3	2	481.90	32.77	486.31	-87.51	481.75	153.19
3	13	1117.88	150.97	1083.71	31.10	1087.30	-89.09
4	9	2406.92	85.19	2304.61	-41.08	2448.54	205.72
4	10	222.28	-82.82	207.53	149.49	227.67	36.30
4	11	870.98	-76.49	789.42	154.49	896.13	42.18
5	2	1442.17	188.22	1446.47	68.47	1442.54	-51.74
6	7	100.36	-0.81	113.54	241.73	99.91	121.76
7	6	339.94	106.10	338.13	-11.59	335.60	226.24
7	8	1019.48	97.43	934.27	-34.31	1019.69	215.89
7	12	591.12	-80.30	549.76	151.35	608.86	40.39
8	7	1021.62	80.46	1081.15	-28.50	1014.71	201.96
8	9	1183.11	260.76	1119.77	127.79	1221.55	22.43
9	4	2005.44	264.38	1910.36	136.77	2046.77	25.03
9	8	1901.26	83.89	1810.97	-43.96	1941.50	204.89
10	4	564.82	92.43	550.92	-30.61	570.40	212.13
10	11	3316.35	90.90	3285.86	-30.18	3310.28	210.99
11	4	2564.26	94.12	2496.12	-29.05	2592.04	213.76
11	10	1409.37	-87.02	1379.65	150.50	1403.81	33.18
11	12	1031.43	89.15	1001.98	-35.82	1046.56	209.85
12	7	769.48	97.21	729.98	-29.25	786.93	217.79
12	11	768.90	269.12	741.18	142.42	784.07	30.05
13	1	1740.04	233.56	1742.25	114.77	1735.89	-4.94
13	2	686.67	19.00	688.15	259.37	681.54	139.73
13	3	996.20	-0.06	967.33	241.07	968.52	120.78
13	14	688.11	49.93	689.63	-69.87	686.70	170.54
14	2	474.99	29.89	476.41	-89.65	470.47	150.70
14	13	487.88	207.65	488.40	87.96	483.92	-31.66

It is obvious from these tables that the phase voltages, phase currents and power flow are significantly unbalanced. Note that the earth return current are significantly higher when the loads are unbalanced as can be seen in Tables 3.22 and 3.23. That is, the more unbalanced the loads in the network, the higher the neutral and ground conductor currents and the higher the earth return current.

Table 3.22 Earth return current in Amp. flowing in the network (mutual impedances neglected) - unbalanced loads

BUS #		Mag.	Angle(°)
1	3	34.2606	-42.34
1	13	81.2150	-46.09
2	3	6.3264	57.19
2	5	7.4049	-56.52
2	13	38.5317	-45.42
2	14	50.0209	-43.79
3	1	34.2126	137.69
3	2	6.3731	238.03
3	13	200.6188	-51.48
4	9	397.0195	180.65
4	10	189.7002	119.58
4	11	955.8804	124.20
5	2	7.2328	123.62
6	7	23.4725	-75.67
7	6	23.5576	107.44
7	8	199.7120	202.22
7	12	98.7672	57.14
8	7	191.5555	22.62
8	9	287.2029	36.32
9	4	396.1934	0.59
9	8	289.6555	216.11
10	4	189.7086	-60.56
10	11	705.4502	219.17
11	4	955.7126	-55.94
11	10	703.2253	39.30
11	12	115.4951	209.67
12	7	99.1637	236.89
12	11	115.0226	29.78
13	1	81.1162	133.93
13	2	38.3810	134.64
13	3	199.7655	128.54
13	14	3.0332	-84.03
14	2	49.8038	136.28
14	13	2.4985	90.37

Table 3.23 Earth return current in Amp. flowing in the network (mutual impedances included) - unbalanced loads

BUS #		Mag.	Angle(°)
1	3	43.6331	-43.88
1	13	54.5865	-45.95
2	3	9.8529	62.86
2	5	8.8420	-65.75
2	13	14.2683	99.14
2	14	11.8818	113.41
3	1	43.5736	136.15
3	2	9.9275	243.51
3	13	41.1717	152.96
4	9	371.9048	211.74
4	10	43.9383	29.07
4	11	210.5424	29.37
5	2	8.6176	114.24
6	7	21.3789	243.73
7	6	20.6760	67.23
7	8	265.8027	205.20
7	12	130.5746	38.58
8	7	256.8489	25.96
8	9	380.8069	29.76
9	4	371.0417	31.81
9	8	383.5789	209.57
10	4	44.3056	208.76
10	11	77.6018	212.84
11	4	212.3812	209.06
11	10	75.8724	34.14
11	12	134.3951	217.37
12	7	131.0912	218.43
12	11	134.0261	37.51
13	1	54.5146	134.08
13	2	14.4029	-80.67
13	3	41.8338	-27.59
13	14	12.2407	-83.21
14	2	12.0934	-66.54
14	13	11.7377	96.34

Note the significant earth current flowing between bus 10 and 11 for the two tables above. When mutual impedances are neglected the earth currents are significantly higher because there are no return paths for the unbalanced currents, hence all the currents flow through earth. The high currents are mainly due to the small series line and mutual impedances of line 10-11 and the large self and mutual shunt admittances at buses 10 and 11.

Tables 3.24 to 3.28 gives the load flow solutions for heavily unbalanced three-phase loads.

Table 3.24 Phase voltages in p.u. for heavily unbalanced loads (mutual impedances included) - unbalanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.0	1.06000	240.00	1.06000	120.00
2	1.08764	-2.50	1.09520	237.86	1.09379	117.37
3	1.08481	-1.99	1.09112	238.38	1.08992	117.94
4	1.02166	-0.70	1.02124	239.28	1.02114	119.25
5	1.08803	-3.86	1.09574	236.49	1.09414	116.00
6	1.03847	-1.20	1.03738	238.71	1.03729	118.66
7	1.03684	-1.21	1.03582	238.70	1.03564	118.65
8	1.03679	-1.20	1.03575	238.70	1.03563	118.65
9	1.03282	-0.99	1.03216	238.94	1.03200	118.91
10	1.01972	-0.68	1.01944	239.30	1.01915	119.27
11	1.02002	-0.69	1.01972	239.29	1.01943	119.26
12	1.02745	-0.99	1.02687	238.97	1.02639	118.92
13	1.08682	-1.78	1.09332	238.64	1.09196	118.15
14	1.08692	-2.14	1.09370	238.26	1.09212	117.75

Table 3.25 Earth return current in Amp. flowing in the network (mutual impedances included) - unbalanced loads

BUS #		Mag.	Angle (°)
1	3	127.9023	32.53
1	13	217.8619	36.13
2	3	26.5658	51.55
2	5	4.5158	213.09
2	13	78.9914	50.05
2	14	96.6130	67.13
3	1	127.7755	212.55
3	2	26.2695	231.98
3	13	243.2932	52.16
4	9	224.5017	168.10
4	10	32.9818	1.69
4	11	144.2935	7.27
5	2	5.0034	31.30
6	7	14.1048	251.69
7	6	14.7064	75.14
7	8	102.8488	245.45
7	12	86.2352	9.18
8	7	105.7140	68.68
8	9	243.5152	-14.47
9	4	223.9793	-11.78
9	8	245.2028	165.27
10	4	33.1093	181.06
10	11	238.0640	158.89
11	4	144.7686	186.53
11	10	236.7148	-20.66
11	12	107.7869	148.42
12	7	86.4329	188.90
12	11	107.4510	-31.43
13	1	217.6400	216.15
13	2	78.6075	230.21
13	3	241.0621	232.48
13	14	40.5641	173.65
14	2	96.1770	247.40
14	13	42.3981	-5.51

Table 3.26 Line flow in MVA in each phase for heavily unbalanced loads (mutual impedances included) - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	17.9131	-47.39	17.5047	-59.27	19.5876	-51.18
1	13	24.0972	-52.55	23.7617	-66.08	26.5425	-56.16
2	3	5.9856	159.18	6.3453	152.55	6.8120	154.88
2	5	21.9415	-16.41	22.4295	-16.66	22.4500	-16.25
2	13	9.3045	170.62	10.2169	166.45	10.2255	166.62
2	14	5.7101	170.06	6.4994	163.57	6.1604	162.01
3	1	17.6344	132.67	17.1956	120.70	19.3941	128.49
3	2	6.5392	-31.00	7.0191	-36.50	7.4335	-33.75
3	13	16.3835	207.08	19.5121	202.22	16.9422	206.25
4	9	33.5819	-79.82	34.0770	-75.60	34.1058	-74.92
4	10	1.8174	77.17	1.5297	78.73	1.9511	77.75
4	11	6.0561	62.06	4.8491	58.06	6.9143	62.69
5	2	21.4442	167.39	21.9183	167.07	21.9503	167.47
6	7	1.4997	-2.42	1.7155	-10.18	1.6779	-4.38
7	6	4.8145	251.89	4.6439	248.71	4.8006	249.63
7	8	13.6679	269.06	13.8816	-87.59	13.9302	268.04
7	12	7.7578	86.82	7.7531	93.57	7.9489	92.06
8	7	15.1292	-89.15	14.8787	267.75	14.8139	-88.16
8	9	15.7502	109.56	16.4532	117.71	16.8724	117.99
9	4	28.3269	101.91	28.9306	106.85	28.9793	107.64
9	8	25.5984	-77.99	25.9140	-72.69	26.3060	-72.34
10	4	6.6115	266.52	6.3321	267.31	6.7408	266.50
10	11	45.8784	268.12	45.4565	-88.80	45.4836	-88.40
11	4	29.5235	264.50	28.2555	264.80	30.3180	264.01
11	10	19.0943	85.47	18.6682	92.91	18.7198	93.88
11	12	14.0122	-85.17	14.0869	-82.09	14.2427	-79.65
12	7	10.2413	267.81	10.2353	-87.08	10.4276	-88.21
12	11	10.4148	96.20	10.5188	100.32	10.7089	103.51
13	1	23.6485	127.60	23.2773	113.87	26.2244	123.58
13	2	9.6330	-17.23	10.6315	-20.52	10.6350	-20.33
13	3	14.6130	-2.22	18.0989	-2.26	15.2230	-2.27
13	14	10.0238	-49.55	10.5775	-50.96	11.0089	-49.04
14	2	6.4086	-28.48	7.3766	-32.15	7.0965	-34.20
14	13	7.2108	153.80	7.5989	150.68	8.1365	151.86

Table 3.27 Line current in Amps. in each phase for heavily unbalanced loads (mutual impedances included) - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	3	1224.57	47.39	1196.66	-60.73	1339.05	171.18
1	13	1647.34	52.55	1624.40	-53.92	1814.50	176.16
2	3	398.79	198.32	419.83	85.31	451.29	-37.51
2	5	1461.85	13.91	1484.05	254.52	1487.31	133.62
2	13	619.91	186.88	676.00	71.41	677.44	-49.24
2	14	380.44	187.44	430.03	74.29	408.13	-44.63
3	1	1177.95	225.34	1142.00	117.68	1289.43	-10.55
3	2	436.81	29.02	466.15	-85.12	494.22	151.69
3	13	1094.39	150.93	1295.84	36.16	1126.41	-88.31
4	9	2381.89	79.12	2417.98	-45.12	2420.26	194.17
4	10	128.90	-77.87	108.54	160.55	138.46	41.50
4	11	429.55	-62.76	344.08	181.22	490.66	56.56
5	2	1428.20	188.75	1449.51	69.42	1453.74	-51.47
6	7	104.65	1.22	119.83	248.89	117.21	123.04
7	6	336.48	106.90	324.88	-10.01	335.89	229.02
7	8	955.23	89.73	971.13	-33.71	974.69	210.61
7	12	542.18	-88.03	542.39	145.13	556.18	26.59
8	7	1057.41	87.95	1040.95	-29.05	1036.54	206.81
8	9	1100.81	249.24	1151.11	120.98	1180.57	0.66
9	4	1987.44	257.10	2031.10	132.09	2034.83	11.26
9	8	1796.00	77.00	1819.31	-48.37	1847.13	191.25
10	4	469.82	92.80	450.10	-28.01	479.29	212.77
10	11	3260.22	91.19	3231.13	-31.90	3233.98	207.67
11	4	2097.40	94.81	2007.90	-25.51	2155.09	215.25
11	10	1356.49	-86.17	1326.60	146.38	1330.66	25.38
11	12	995.45	84.48	1001.04	-38.62	1012.41	198.91
12	7	722.30	91.21	722.28	-33.95	736.20	207.12
12	11	734.53	262.81	742.28	138.65	756.05	15.40
13	1	1576.76	230.62	1542.79	124.77	1740.29	-5.43
13	2	642.28	15.45	704.64	259.16	705.75	138.48
13	3	974.32	0.44	1199.57	240.90	1010.22	120.43
13	14	668.34	47.77	701.06	-70.41	730.56	167.19
14	2	427.26	26.33	488.74	-89.60	470.86	151.95
14	13	480.74	204.06	503.47	87.58	539.87	-34.11

Table 3.28 Return current in neutral and ground conductors in Amps. for heavily unbalanced loads (mutual impedances included)

BUS #		n		g	
I	J	Mag.	Angle(°)	Mag.	Angle(°)
1	3	8.9830	199.86	8.9830	199.86
1	13	15.3006	203.46	15.3006	203.46
2	3	1.8703	222.29	1.8703	222.29
2	5	0.3171	20.42	0.3171	20.42
2	13	5.5613	220.79	5.5613	220.79
2	14	6.7853	234.46	6.7853	234.46
3	1	8.9741	19.88	8.9741	19.88
3	2	1.8494	42.72	1.8494	42.72
3	13	17.0851	219.49	17.0851	219.49
4	9	15.7665	-24.57	15.7665	-24.57
4	10	2.3161	169.02	2.3161	169.02
4	11	10.1263	174.60	10.1263	174.60
5	2	0.3514	198.63	0.3514	198.63
6	7	1.4416	46.12	1.4416	46.12
7	6	1.5031	229.58	1.5031	229.58
7	8	10.5134	39.88	10.5134	39.88
7	12	7.2541	166.47	7.2541	166.47
8	7	10.8062	223.12	10.8062	223.12
8	9	24.8803	139.96	24.8803	139.96
9	4	15.7299	155.54	15.7299	155.54
9	8	25.0527	-40.30	25.0527	-40.30
10	4	2.3251	-11.62	2.3251	-11.62
10	11	24.2373	-46.70	24.2373	-46.70
11	4	10.1597	-6.14	10.1597	-6.14
11	10	24.1000	133.75	24.1000	133.75
11	12	11.0156	-57.15	11.0156	-57.15
12	7	7.2707	-13.81	7.2707	-13.81
12	11	10.9812	123.01	10.9812	123.01
13	1	15.2850	23.48	15.2850	23.48
13	2	5.5342	40.95	5.5342	40.95
13	3	16.9285	39.81	16.9285	39.81
13	14	7.3501	-33.63	7.3501	-33.63
14	2	6.7547	54.72	6.7547	54.72
14	13	7.6824	147.20	7.6824	147.20

3.6 Conclusions

The development of a Gauss-Seidel load flow algorithm is described in this chapter. The algorithm permits the analysis of an unbalanced power system network in the phase frame of reference. As such, the algorithm forms the basis of a computer program which can be used to establish a three-phase load flow solution. The influence of shunt and series compensation in both balanced and unbalanced forms could be established together with different methods of voltage control, tap staggering and unbalanced load conditions.

This chapter has presented the results of an unbalanced load flow program which included the mutual coupling terms and the neutral and ground conductors in the load flow algorithm to demonstrate several important points that must be considered in the analysis of distribution systems. The results of the unbalanced load flow program are compared with those obtained from a balanced load flow model [27].

Many distribution feeders and networks contain many miles of primary and secondary lines with many loads (e.g., networks containing over 1000 nodes are common in practice). In the past, many load models assumed the network loads were balanced or lumped together to form a composite load and/or the three-phase primary and secondary lines and cables were transposed and/or the neutral and ground conductors were omitted from the model. In many cases, these assumptions provided meaningful results that were and still are used in distribution planning studies.

One of the objectives of this research was to reveal that, even if a network's loads are balanced three-phase loads, the phase currents and the power flowing throughout the network can be unbalanced, resulting in significant current flowing in the neutral conductor, ground conductor and earth. This conclusion was based on a load flow model that included the mutual coupling terms between the various phases in the network (i.e., the three-phase conductors, the neutral conductor and the ground conductor).

Unbalanced load flow studies for planning and operating studies of distribution systems are becoming more and more important for utilities. The unbalanced load flow studies provide a means of estimating the magnitude of the neutral and ground currents that are flowing in a given distribution network configuration. Excessive magnitudes of these currents will cause false trips of ground fault overcurrent protective devices in the system. In addition, these studies will reveal inadequate system and load grounding practices in distribution systems (e.g., ground loop currents, stray neutral current, etc.) which cannot be revealed with a balanced load flow model. The unbalanced load flow studies provide a significantly better estimate of the system losses in the primary lines (i.e., including the losses that occur in the neutral and ground conductors) than balanced system methodologies. Considerable research is being undertaken to decrease the computational time of the unbalanced load flow algorithm particularly for large systems (e.g., 1000-10,000 buses).

One of the disadvantages of the unbalanced load flow program is that it requires significantly more computational time (e.g., 2 to 3 times that of Newton Raphson method). More time is required to prepare a data base for a distribution network in which the details of the line construction, transposition cycles, load connections, grounding connections, ground conditions, etc. are accounted for in the load flow model equations. This increase in computational time can be justified for many case studies in assessing the electrical characteristics which are not accounted for by "balanced load flow methods". The unbalanced load flow programs provide an excellent means for distribution planning and for operating engineers to balance their loads and to assess the impact of unbalanced loading patterns on changing distribution network operating configurations. This research clearly revealed the impact on the phase voltages, the power flow and the return current in a network when the mutual coupling terms and the neutral and ground conductors are included in the load flow methodology even when the network loads are balanced three-phase loads. It also reveals the impact of unbalanced network loads on the phase voltages, line flow and the ground return current. The more unbalanced the network loads (e.g., single phase network loads) the more severe the unbalance in the line flow and bus voltages, and the higher the earth return current at the buses.

CHAPTER IV LOAD FLOW / RELIABILITY STUDY

4.1 Introduction

An example relating the load flow analysis to the reliability indices is shown in this chapter. Note that the data used in this example is purely arbitrary for illustration purposes. The input data for the two algorithms proposed is not program dependent, i.e., it allows the user to use any data he or she chooses. The load point # 3 for the 5 bus test system shown in Figure 4.1 is used for illustrating the relationship between maintenance, network operating conditions and the reliability levels of the customer.

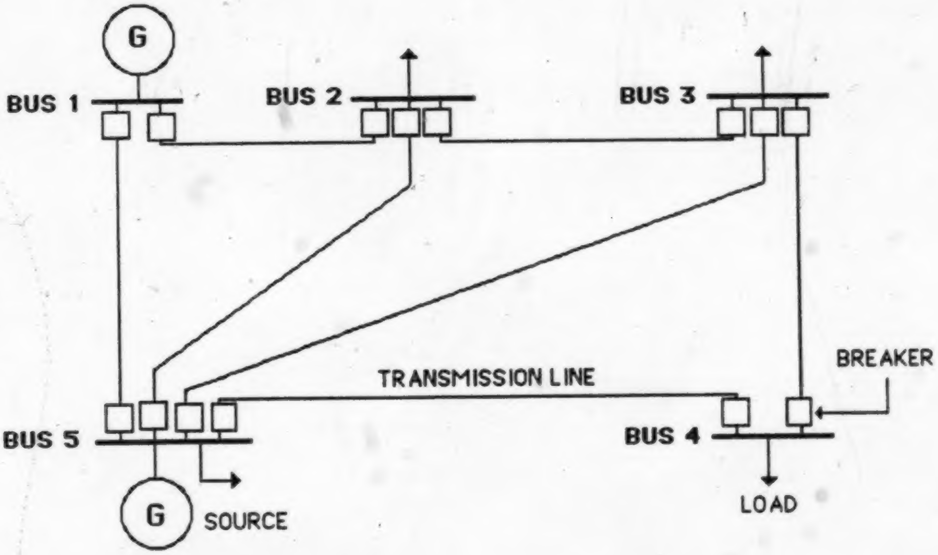


Figure 4.1(a) 5-bus test system single line diagram

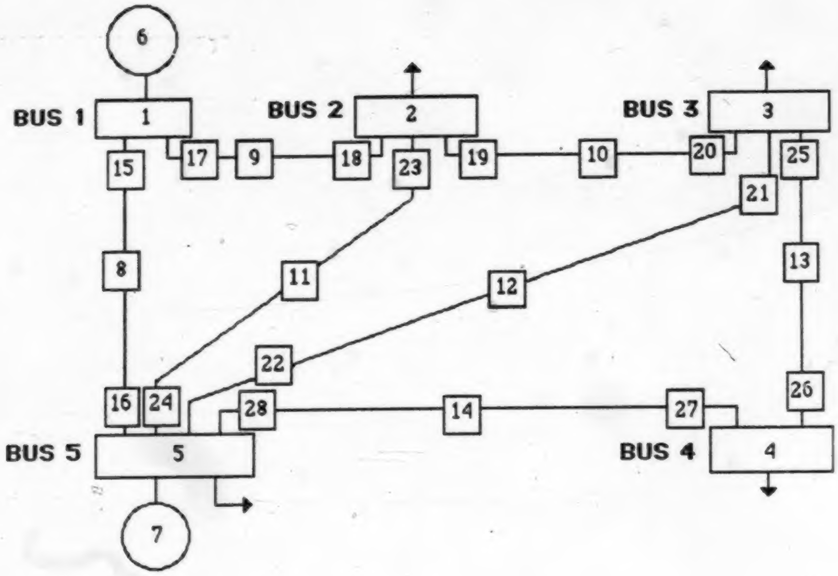


Figure 4.1(b) 5-bus test system element block diagram.

4.2 Test Criterion

Consider performing maintenance on source element #7 for a duration of five hours starting on May 1, at 16: 00 hours. Assume that the circuit breakers will trip when the neutral conductor's current exceeds 1.0 Amp. and the mutual impedances of the lines are included in the load flow analysis in the example.

4.3 Test Results

The operational paths to load point #3 are shown in Figure 4.2. The frequency of interruptions when no maintenance activity is estimated for the three cases described in Chapter II (i.e., weather independent, repair/maintenance activities are continued in adverse weather, repair/maintenance activities are discontinued in adverse weather, and switching capability for each of the three cases) are shown in Tables 4.1, 4.2 and 4.3. Note that only the hourly results around the maintenance duration period are shown in the tables to illustrate the impact of maintenance on the level of frequency of interruptions experienced by load point #3. The results prior to the maintenance activities are of no interest since the network configuration does not change and hence these results will not be shown.

Path #	Components forming path
1	7 5 22 12 21 3
2	6 1 17 9 18 2 19 10 20 3
3	7 5 28 12 27 4 26 13 25 3
4	7 5 23 11 23 2 19 10 20 3
5	6 1 15 8 16 5 22 12 21 3
6	6 1 17 9 18 2 23 11 24 5 22 12 21 3
7	7 5 16 8 15 1 17 9 18 2 19 10 20 3
8	6 1 15 8 12 5 28 14 27 4 26 13 25 3
9	6 1 15 8 16 5 24 11 23 2 19 10 20 3
10	6 1 17 9 18 2 23 11 24 5 28 14 27 4 26 13 25 3

Figure 4.2 Operational paths to load point #3.

Table 4.1 Frequency of interruptions experienced by load point #3 - weather independent

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01045275	0.01041562
0.33299100	0.01045634	0.01041919
0.33310503	0.01045992	0.01042276
0.33321899	0.01046351	0.01042634
0.33333302	0.01046709	0.01042991
0.33344698	0.01047069	0.01043348
0.33356202	0.01047426	0.01043705
0.33367598	0.01047783	0.01044062
0.33379000	0.01048143	0.01044419
0.33390403	0.01048501	0.01044776
0.33401799	0.01048860	0.01045134
0.33413202	0.01049218	0.01045490
0.33424699	0.01049576	0.01045848
0.33436102	0.01049934	0.01046205
AVERAGE	~ 0.01047	~ 0.01044

Table 4.2 Frequency of interruptions experienced by load point #3 - repair/maintenance activities continued in adverse weather

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01276070	0.01270643
0.33299100	0.01276507	0.01271079
0.33310503	0.01276945	0.01271515
0.33321899	0.01277385	0.01271953
0.33333302	0.01277822	0.01272388
0.33344698	0.01278260	0.01272823
0.33356202	0.01278697	0.01273259
0.33367598	0.01279135	0.01273694
0.33379000	0.01279573	0.01274130
0.33390403	0.01280010	0.01274565
0.33401799	0.01280449	0.01275001
0.33413202	0.01280886	0.01275437
0.33424699	0.01281324	0.01275873
0.33436102	0.01281762	0.01276309
AVERAGE	~ 0.01279	~ 0.01274

Table 4.3 Frequency of interruptions experienced by load point #3 - repair/maintenance activities discontinued in adverse weather

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01277450	0.01272016
0.33299100	0.01277888	0.01272452
0.33310503	0.01278326	0.01272889
0.33321899	0.01278767	0.01273327
0.33333302	0.01279205	0.01273764
0.33344698	0.01279643	0.01274200
0.33356202	0.01280081	0.01274636
0.33367598	0.01280519	0.01275072
0.33379000	0.01280957	0.01275508
0.33390403	0.01281395	0.01275944
0.33401799	0.01281834	0.01276382
0.33413202	0.01282272	0.01276818
0.33424699	0.01282710	0.01277254
0.33436102	0.01283149	0.01277691
AVERAGE	~ 0.01280	~ 0.01275

The load flow solutions (e.g., phase voltages, power flow, etc.) with no maintenance activity are shown in Table 4.4 to Table 4.8 for balanced three-phase loads, and Tables 4.9 to 4.13 for unbalanced three-phase loads.

Table 4.4 Phase voltages in p.u. at each bus - balanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.00	1.06000	240.00	1.06000	120.00
2	1.06605	0.58	1.06600	240.57	1.06595	120.57
3	1.06837	0.71	1.06828	240.70	1.06823	120.70
4	1.06946	0.78	1.06936	240.76	1.06931	120.77
5	1.06944	0.79	1.06934	240.78	1.06929	120.78

Table 4.5 Line flow in MVAR in phase a, b and c - balanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	2	4.8750	195.38	4.7789	195.77	4.8001	195.32
1	5	9.6409	198.70	9.4923	198.89	9.5317	198.57
2	1	4.8992	15.80	4.8030	16.21	4.8238	15.75
2	3	2.0173	214.17	1.9977	213.75	2.0084	213.72
2	5	4.2615	207.48	4.2306	207.03	4.2501	207.03
3	2	2.0111	33.86	1.9937	33.52	2.0037	33.46
3	4	1.0977	213.15	1.0906	212.47	1.0965	212.58
3	5	1.5943	202.59	1.5885	202.05	1.5940	202.13
4	3	1.0874	32.28	1.0826	31.77	1.0879	31.83
4	5	0.9913	165.47	0.9911	165.53	0.9878	165.25
5	1	9.7204	19.38	9.5707	19.58	9.6098	19.26
5	2	4.2616	27.34	4.2333	26.95	4.2520	26.94
5	3	1.5845	21.65	1.5811	21.31	1.5858	21.32
5	4	1.0271	-20.88	1.0186	-19.59	1.0185	-20.29

Table 4.6 Line current flowing in Amp. in phase a, b and c - balanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	2	333.268	164.62	326.699	44.23	328.145	-75.32
1	5	659.070	161.30	648.910	41.11	651.610	-78.57
2	1	333.015	-15.22	326.493	224.36	327.925	104.82
2	3	137.121	146.41	135.800	26.82	136.531	266.85
2	5	289.667	153.11	287.584	33.54	288.923	-86.46
3	2	136.403	-33.15	135.239	207.18	135.920	87.24
3	4	74.456	147.56	73.978	28.23	74.383	268.13
3	5	108.139	158.12	107.754	38.65	108.130	-81.43
4	3	73.682	-31.51	73.364	208.99	73.722	88.94
4	5	67.166	195.31	67.158	75.24	66.939	-44.48
5	1	658.639	-18.59	648.563	221.20	651.238	101.53
5	2	288.759	-26.55	286.867	213.82	288.147	93.85
5	3	107.360	-20.86	107.144	219.47	107.465	99.46
5	4	69.592	21.67	69.027	260.37	69.020	141.07

Table 4.7 Current in Amp. flowing in neutral and ground conductors - balanced loads

BUS #		n		g	
I	J	Mag.	Angle(°)	Mag.	Angle(°)
1	2	1.3535	-0.46	1.3160	-0.40
1	5	2.6172	-3.13	2.5654	-3.00
2	1	1.3521	180.79	1.3148	180.68
2	3	0.5313	-15.92	0.5244	-15.19
2	5	1.0946	-13.00	1.0900	-12.20
3	2	0.5260	165.14	0.5202	165.68
3	4	0.2828	-15.53	0.2811	-14.54
3	5	0.4026	-8.16	0.4016	-7.24
4	3	0.2776	193.35	0.2769	192.76
4	5	0.2439	30.80	0.2464	30.56
5	1	2.6149	177.10	2.5634	177.19
5	2	1.0886	167.80	1.0851	168.46
5	3	0.3976	185.79	0.3976	185.31
5	4	0.2753	223.72	0.2704	221.28

Table 4.8 Earth current in Amp. - balanced loads

BUS #		Mag.	Angle(°)
1	2	6.8482	189.91
1	5	11.3851	182.05
2	1	6.8101	11.40
2	3	1.8950	144.51
2	5	3.5639	145.06
3	2	1.6890	-30.50
3	4	0.9664	130.86
3	5	1.1730	138.31
4	3	0.7269	-40.75
4	5	0.3334	189.11
5	1	11.3104	3.26
5	2	3.2664	-31.04
5	3	0.8427	-28.83
5	4	1.3946	96.82

Table 4.9 Phase voltages in p.u. at each bus - unbalanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.00	1.06000	240.00	1.06000	120.00
2	1.06631	0.60	1.06651	240.60	1.06653	120.61
3	1.06869	0.73	1.06882	240.73	1.06916	120.76
4	1.06988	0.80	1.06994	240.80	1.07000	120.81
5	1.06977	0.81	1.06983	240.81	1.06993	120.82

Table 4.10 Line flow in MVAR in phase a, b and c - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	2	5.0271	195.73	5.0746	196.42	5.1530	196.04
1	5	9.9220	198.91	9.9085	199.20	10.0711	198.94
2	1	5.0532	16.17	5.1026	16.90	5.1813	16.53
2	3	2.0873	213.84	2.0203	213.94	2.3376	213.02
2	5	4.3611	207.41	4.2291	206.99	4.3128	207.00
3	2	2.0814	33.54	2.0163	33.72	2.3345	32.83
3	4	1.1850	213.31	1.1524	211.77	0.8321	213.98
3	5	1.5973	202.84	1.5568	201.42	1.1873	201.13
4	3	1.1748	32.51	1.1448	31.11	0.8226	32.97
4	5	0.7420	143.11	0.7215	142.29	0.8164	152.70
5	1	10.0071	19.61	9.9952	19.93	10.1598	19.68
5	2	4.3619	27.28	4.2317	26.92	4.3151	26.92
5	3	1.5873	21.90	1.5495	20.66	1.1789	19.98
5	4	0.8179	-43.49	0.7826	-43.16	0.8678	-33.28

Table 4.11 Line current flowing in Amp. in phase a, b and c - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	2	343.660	164.27	346.910	43.58	352.267	-76.04
1	5	678.289	161.09	677.366	40.80	688.480	-78.94
2	1	343.402	-15.57	346.696	223.70	352.035	104.09
2	3	141.847	146.76	137.267	26.66	158.822	267.60
2	5	296.371	153.19	287.346	33.61	293.027	-86.39
3	2	141.134	-32.81	136.703	207.01	158.221	87.93
3	4	80.348	147.42	78.134	28.96	56.394	266.78
3	5	108.306	157.89	105.548	39.31	80.474	-80.36
4	3	79.569	-31.71	77.532	209.68	55.710	87.84
4	5	50.253	217.70	48.868	98.51	55.288	-31.89
5	1	677.854	-18.80	677.012	220.88	688.100	101.15
5	2	295.464	-26.47	286.630	213.89	292.251	93.91
5	3	107.518	-21.09	104.955	220.15	79.842	100.84
5	4	55.405	44.30	53.012	-76.03	58.777	154.11

Table 4.12 Current in Amp. flowing in neutral and ground conductors - unbalanced loads

BUS #		n		g	
I	J	Mag.	Angle(°)	Mag.	Angle(°)
1	2	1.2458	14.13	1.2533	9.67
1	5	2.5456	5.87	2.5395	3.17
2	1	1.2462	194.50	1.2531	189.97
2	3	0.7914	30.63	0.6702	20.67
2	5	1.2760	-11.69	1.2183	-11.44
3	2	0.7948	211.39	0.6714	201.42
3	4	0.8056	-72.81	0.6027	-65.17
3	5	0.9160	-67.62	0.6936	-58.62
4	3	0.7938	107.27	0.5932	115.02
4	5	0.5030	68.16	0.4009	65.81
5	1	2.5449	186.11	2.5384	183.37
5	2	1.2703	169.00	1.2136	169.15
5	3	0.8992	112.62	0.6806	121.80
5	4	0.5607	251.65	0.4467	249.65

Table 4.13 Earth current in Amp. - unbalanced loads

BUS #		Mag.	Angle (°)
1	2	4.9723	225.02
1	5	7.5588	226.04
2	1	5.0446	46.91
2	3	42.5569	155.28
2	5	0.7961	226.93
3	2	42.3805	-24.47
3	4	46.9995	-25.06
3	5	60.4007	-21.75
4	3	47.1772	154.70
4	5	0.8606	-58.63
5	1	7.6730	47.69
5	2	1.0282	65.43
5	3	60.6599	157.97
5	4	2.3425	112.92

Now assuming that element #7 is under maintenance, i.e., no generation on bus #5. The load flow results for the new configuration are shown in Table 4.14 to Table 4.18 for balanced three-phase loads, and Table 4.19 to Table 4.23 for unbalanced three-phase loads.

Table 4.14 Phase voltages in p.u. at each bus - balanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.00	1.06000	240.00	1.06000	120.00
2	1.05345	-0.12	1.05351	239.87	1.05339	119.87
3	1.05300	-0.16	1.05208	239.83	1.05294	119.83
4	1.05309	-0.15	1.05315	239.83	1.05302	119.83
5	1.05286	-0.14	1.05292	239.84	1.05279	119.84

Table 4.15 Line flow in MVAR in phase a, b and c - balanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	2	2.7025	58.31	2.7229	55.74	2.7555	56.57
1	5	4.1954	55.98	4.2337	53.43	4.2824	54.30
2	1	2.6974	238.35	2.7151	235.74	2.7483	236.58
2	3	0.5443	18.68	0.9590	52.22	0.5578	18.56
2	5	0.6373	34.08	0.6525	32.22	0.6552	33.29
3	2	0.5502	200.46	0.9692	232.71	0.5628	200.05
3	4	0.1023	213.94	0.7235	258.50	0.0989	212.96
3	5	0.2563	141.53	0.7505	243.75	0.2555	141.08
4	3	0.0925	23.34	0.7081	78.23	0.0905	23.50
4	5	0.8847	112.59	0.8798	113.10	0.8816	112.87
5	1	4.1829	235.98	4.2176	233.39	4.2668	234.27
5	2	0.6532	216.11	0.6645	213.84	0.6686	215.02
5	3	0.2756	-43.28	0.7301	62.94	0.2724	-43.12
5	4	0.9907	-69.93	0.9642	-69.02	0.9731	-69.39

Table 4.16 Line current flowing in Amp. in phase a, b and c - balanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	2	184.751	-58.31	186.143	184.26	188.372	63.43
1	5	286.804	-55.98	289.423	186.57	292.756	65.70
2	1	185.546	121.54	186.757	4.13	189.056	243.29
2	3	37.441	-18.80	65.964	187.64	38.370	101.31
2	5	43.836	-34.20	44.884	207.65	45.073	86.58
3	2	37.865	159.38	66.756	7.12	38.731	-80[illegible]
3	4	7.041	145.90	49.835	-18.67	6.809	266.87
3	5	17.636	218.32	51.695	-3.93	17.584	-21.25
4	3	6.362	-23.49	48.722	161.59	6.229	96.33
4	5	60.875	247.25	60.533	126.72	60.665	6.97
5	1	287.889	123.88	290.265	6.45	293.683	245.57
5	2	44.955	143.75	45.733	26.00	46.020	264.82
5	3	18.971	43.14	50.243	176.89	18.747	162.96
5	4	68.184	69.79	66.355	-51.15	66.981	189.23

Table 4.17 Current in Amp. flowing in neutral and ground conductors - balanced loads

BUS #		n		g	
I	J	Mag.	Angle(°)	Mag.	Angle(°)
1	2	0.6190	128.19	0.6363	131.93
2	1	0.6260	-52.14	0.6418	-48.36
1	5	0.9568	130.86	0.9866	134.58
5	1	0.9661	-49.48	0.9938	-45.71
2	3	0.9673	-39.05	0.6023	-42.19
3	2	0.9588	141.36	0.5950	138.30
2	5	0.1424	202.57	0.1493	200.45
5	2	0.1513	-27.85	0.1561	-24.66
3	4	1.1599	144.96	0.7558	144.60
4	3	1.1683	-35.44	0.7629	-35.88
3	5	1.5074	142.73	0.9951	141.29
5	3	1.5197	-37.72	1.0054	-39.23
4	5	0.2374	93.57	0.2344	95.03
5	4	0.3016	268.96	0.2862	267.47

Table 4.18 Earth current in Amp. - balanced loads

BUS #		Mag.	Angle(°)
1	2	10.5224	-73.35
1	5	13.8887	262.09
2	1	10.7031	106.75
2	3	20.3704	258.88
2	5	9.2394	175.30
3	2	20.5916	79.25
3	4	26.2730	96.55
3	5	30.3177	101.26
4	3	26.0097	-83.56
4	5	12.4435	268.16
5	1	14.1102	82.56
5	2	9.0945	-2.53
5	3	29.9253	-78.86
5	4	13.8728	90.39

Table 4.19 Phase voltages in p.u. at each bus - unbalanced loads

BUS #	a		b		c	
	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	1.06000	-0.00	1.06000	240.00	1.06000	120.00
2	1.05371	-0.10	1.05402	239.90	1.05399	119.91
3	1.05333	-0.14	1.05361	239.86	1.05388	119.89
4	1.05351	-0.13	1.05374	239.87	1.05372	119.88
5	1.05320	-0.12	1.05343	239.87	1.05344	119.88

Table 4.20 Line flow in MVAR in phase a, b and c - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	2	2.5687	60.05	2.4596	59.03	2.4485	60.71
1	5	3.9461	57.96	3.8572	56.15	3.8050	58.07
2	1	2.5652	240.11	2.4550	239.06	2.4450	240.76
2	3	0.4730	17.95	0.5390	16.35	0.2400	4.02
2	5	0.5389	35.83	0.6548	32.44	0.5948	34.06
3	2	0.4788	200.01	0.5430	197.77	0.2416	187.76
3	4	0.1889	214.89	0.1608	207.99	0.1672	25.33
3	5	0.2499	142.92	0.2571	132.79	0.3767	62.88
4	3	0.1780	29.47	0.1537	22.55	0.1755	210.57
4	5	1.0115	88.57	1.0118	87.33	0.9490	96.51
5	1	3.9369	237.99	3.8460	236.15	3.7955	238.08
5	2	0.5557	218.18	0.6669	214.05	0.6086	215.95
5	3	0.2688	-42.11	0.2748	-50.54	0.3997	244.56
5	4	1.1252	268.72	1.1027	267.56	1.0470	-84.10

Table 4.21 Line current flowing in Amp. in phase a, b and c - unbalanced loads

BUS #		a		b		c	
I	J	Mag.	Angle(°)	Mag.	Angle(°)	Mag.	Angle(°)
1	2	175.600	-60.05	168.146	180.97	167.382	59.29
1	5	269.766	-57.96	263.688	183.85	260.117	61.93
2	1	176.410	119.80	168.783	0.84	168.097	239.15
2	3	32.527	-18.05	37.055	223.55	16.500	115.89
2	5	37.063	-35.93	45.019	207.46	40.893	85.85
3	2	32.939	159.86	37.348	42.08	16.612	-67.87
3	4	12.993	144.97	11.057	31.87	11.499	94.56
3	5	17.194	216.94	17.680	107.06	25.905	57.01
4	3	12.240	-29.60	10.571	217.32	12.072	269.31
4	5	69.572	-88.70	69.578	152.53	65.259	23.37
5	1	270.876	121.89	264.559	3.72	261.085	241.80
5	2	38.235	141.70	45.873	25.81	41.861	263.94
5	3	18.494	41.99	18.906	-69.59	27.493	235.33
5	4	77.416	91.16	75.855	-27.69	72.022	203.98

Table 4.22 Current in Amp. flowing in neutral and ground conductors - unbalanced loads

BUS #		n		g	
I	J	Mag.	Angle(°)	Mag.	Angle(°)
1	2	0.9395	123.75	0.8411	126.60
1	5	1.3234	121.68	1.2075	126.04
2	1	0.9467	-56.44	0.8468	-53.59
2	3	0.5581	96.18	0.3761	90.19
2	5	0.0685	214.31	0.0387	258.91
3	2	0.5685	264.17	0.3847	269.90
3	4	0.7154	268.48	0.4932	269.92
3	5	0.7534	268.10	0.4996	269.02
4	3	0.7040	91.73	0.4839	90.31
4	5	0.5608	90.22	0.4625	92.66
5	1	1.3334	-58.50	1.2154	-54.15
5	2	0.0770	225.17	0.0519	259.49
5	3	0.7367	92.15	0.4861	91.28
5	4	0.6253	269.34	0.5153	266.73

Table 4.23 Earth current in Amp. - unbalanced loads

BUS #		Mag.	Angle (°)
1	2	13.9444	-64.29
1	5	19.3261	-78.63
2	1	14.1247	115.67
2	3	21.9286	261.63
2	5	7.2553	194.35
3	2	22.1544	81.93
3	4	26.0542	94.72
3	5	29.9338	99.21
4	3	25.7957	-85.41
4	5	12.6046	268.97
5	1	19.5714	101.47
5	2	7.2399	17.29
5	3	29.5487	-80.93
5	4	14.0251	90.97

It can be seen that in Table 4.17 for a balanced load the neutral currents flowing between bus 3 and 4 (element #13) and between bus 3 and 5 (element #14) exceed the 1.0 Amp. limit, causing the circuit breakers (element # 25, 26, 27 and 28) to trip, hence isolating the two lines. The operational paths to load point #3 for the new network configuration are shown in Figure 4.5. The frequency of interruptions is given in Tables 4.24 to 4.26.

Path #	Components forming path
1	6 1 17 9 18 2 19 10 20 3
2	6 1 15 8 16 5 22 12 21 3
3	6 1 17 9 18 2 23 11 24 5 22 12 21 3
4	6 1 15 8 16 5 24 11 23 2 19 10 20 3

Figure 4.3 Operational paths to load point #3 - with element #7 under maintenance and element #13 and #14 overloaded (balanced loads)

Table 4.24 Frequency of interruptions experienced by load point #3, weather independent - with element # 7 under maintenance and element #13 and #14 overloaded (balanced loads)

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01045275	0.01041562
0.33299100	0.01045634	0.01041919
0.33310503	0.01045992	0.01042276
0.33321899	0.01046351	0.01042634
0.33333302	0.01916321	0.01914267
0.33344698	0.01916978	0.01914923
0.33356202	0.01917633	0.01915578
0.33367598	0.01918288	0.01916232
0.33379000	0.01918945	0.01916888
0.33390403	0.01919600	0.01917543
0.33401799	0.01047377	0.01043680
0.33413202	0.01047735	0.01044037
0.33424699	0.01048094	0.01044394
0.33436102	0.01048452	0.01044751
AVERAGE	~ 0.01337	~ 0.01334

Table 4.25 Frequency of interruptions experienced by load point #3, repair/maintenance activities continued in adverse weather - with element #7 under maintenance and element #13 and #14 overloaded (balanced loads)

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01276070	0.01270643
0.33299100	0.01276507	0.01271079
0.33310503	0.01276945	0.01271515
0.33321899	0.01277385	0.01271953
0.33333302	0.02338600	0.02335734
0.33344698	0.02339401	0.02336534
0.33356202	0.02340201	0.02337333
0.33367598	0.02341002	0.02338133
0.33379000	0.02341802	0.02338932
0.33390403	0.02342602	0.02339731
0.33401799	0.01277899	0.01272945
0.33413202	0.01278338	0.01273381
0.33424699	0.01278775	0.01273816
0.33436102	0.01279214	0.01274252
AVERAGE	~ 0.01632	~ 0.01627

Table 4.26 Frequency of interruptions experienced by load point #3 - repair/maintenance activities discontinued in adverse weather - with element #7 under maintenance and element #13 and #14 overloaded (balanced loads)

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01277450	0.01272016
0.33299100	0.01277888	0.01272452
0.33310503	0.01278326	0.01272889
0.33321899	0.01278767	0.01273327
0.33333302	0.02339409	0.02336402
0.33344698	0.02340210	0.02337202
0.33356202	0.02341011	0.02338002
0.33367598	0.02341811	0.02338802
0.33379000	0.02342612	0.02339602
0.33390403	0.02343414	0.02340402
0.33401799	0.01279174	0.01273764
0.33413202	0.01279612	0.01274199
0.33424699	0.01280050	0.01274635
0.33436102	0.01280489	0.01275072
AVERAGE	~ 0.01633	~ 0.01628

From the unbalanced three-phase loads load flow results, it is obvious that circuit breakers #15 and #16 will trip since the neutral current flowing between bus 1 and 5 (element #8) exceeds the 1.0 Amp. limit (see Table 4.20). The operational paths to load point #3 for unbalanced loads case study are as shown in Figure 4.4 and the frequency of interruptions is given in Tables 4.27 to 4.29.

Path #	Components forming path
1	6 1 17 9 18 2 19 10 20 3
2	6 1 17 9 18 2 23 11 24 5 22 12 21 3
3	6 1 17 9 18 2 23 11 24 5 28 14 27 4 26 13 25 3

Figure 4.4 Operational paths to load point #3 - with element #7 under maintenance and element #8 overloaded (unbalanced loads)

Table 4.27 Frequency of interruptions experienced by load point #3, weather independent - with element #7 under maintenance and element #8 overloaded (unbalanced loads)

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01045275	0.01041562
0.33299100	0.01045634	0.01041919
0.33310503	0.01045992	0.01042276
0.33321899	0.01046351	0.01042634
0.33333302	0.06155255	0.06154664
0.33344698	0.06157363	0.06156772
0.33356202	0.06159471	0.06158880
0.33367598	0.06161574	0.06160983
0.33379000	0.06163683	0.06163092
0.33390403	0.06165789	0.06165198
0.33401799	0.01047928	0.01044230
0.33413202	0.01048286	0.01044587
0.33424699	0.01048644	0.01044944
0.33436102	0.01049002	0.01045301
AVERAGE	~ 0.02751	~ 0.02748

Table 4.28 Frequency of interruptions experienced by load point #3, repair/maintenance activities continued in adverse weather - with element #7 under maintenance and element #8 overloaded (unbalanced loads)

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01276070	0.01270643
0.33299100	0.01276507	0.01271079
0.33310503	0.01276945	0.01271515
0.33321899	0.01277385	0.01271953
0.33333302	0.07513148	0.07512325
0.33344698	0.07515723	0.07514900
0.33356202	0.07518297	0.07517475
0.33367598	0.07520866	0.07520044
0.33379000	0.07523441	0.07522613
0.33390403	0.07526010	0.07525182
0.33401799	0.01278571	0.01273616
0.33413202	0.01279008	0.01274052
0.33424699	0.01279446	0.01274487
0.33436102	0.01279885	0.01274923
AVERAGE	~ 0.03357	~ 0.03355

Table 4.29 Frequency of interruptions experienced by load point #3 - repair/maintenance activities discontinued in adverse weather - with element #7 under maintenance and element #8 overloaded (unbalanced loads)

TIME (YEAR)	EXCLUDING SWITCHING	INCLUDING SWITCHING
0.33287698	0.01277450	0.01272016
0.33299100	0.01277888	0.01272452
0.33310503	0.01278326	0.01272889
0.33321899	0.01278767	0.01273327
0.33333302	0.07513487	0.07512617
0.33344698	0.07516050	0.07515192
0.33356202	0.07518625	0.07517767
0.33367598	0.07521194	0.07520336
0.33379000	0.07523769	0.07522905
0.33390403	0.07526338	0.07525474
0.33401799	0.01279845	0.01274435
0.33413202	0.01280283	0.01274870
0.33424699	0.01280721	0.01275306
0.33436102	0.01281160	0.01275743
AVERAGE	~ 0.03359	~ 0.03357

4.4 Discussion/Conclusions

Comparing the results of Tables 4.24 to 4.29 to those of Tables 4.1 to 4.3, it is clear that maintenance activities can significantly affect the reliability indices (e.g., frequency of interruptions) of the customers. Therefore, it is necessary to schedule the maintenance activities carefully so as to avoid overloading the distribution or transmission lines causing further removal of components (e.g., isolating the overloaded components) from the network, hence affecting the load point frequency of interruptions. For example, consider the weather independent excluding switching case, the frequency of interruptions is ~ 0.01047 with no maintenance activities, ~ 0.01337 for element #7 under maintenance and a balanced load, and ~ 0.02751 for element #7 under maintenance and an unbalanced load. If the system is unbalanced at the time of maintenance, the system shown in Figure 4.1 is more likely to experience outages than when performing maintenance activities while the system is balanced or approximately balanced. Hence, in order to achieve an effective maintenance schedule a detailed load flow solution of the network configuration is necessary to provide the utilities with a more accurate load distribution of the system. Note also that the severity of the maintenance activities on the frequency of interruptions and load flow solutions depends on the components being maintained, the weather conditions during the maintenance activities, and the overload limits of the breakers (as in Tables 4.24 to 4.26 and Tables 4.27 to 4.29).

CHAPTER V CONCLUSIONS

5.1 Introduction

The objective of distribution network operation and maintenance is to ensure that the growing demand for electricity, in terms of increasing growth rates and high load densities, can be satisfied in an optimum way by the planning, operation and maintenance of systems. In planning a distribution system, the load magnitude and its geographic location must be determined in order to serve the load at its maximum cost effectiveness by minimizing feeder losses and construction costs, while considering the constraints of service reliability. Since the distribution system is directly connected to the customers, its failure affects the customers more directly than transmission or generating systems. The demand, type, load factor, and other customer characteristics dictate the type of design, planning, operation and maintenance of the distribution serving them. In order to have continuity and high quality of supply, the utility will have to know the type of network configuration it has, i.e., knowing the type of operation and maintenance its distribution systems require.

This thesis proposes two algorithms to determine the operational characteristics of the three-phase distribution network configuration and the service reliability levels of the network and its customers. The reliability algorithm uses the Weibull probability distribution function, which allows the rate parameters to be varied with respect to operating time, representing the life distribution of the system components in evaluating the reliability indices over a given operating time period. The impact of scheduled maintenance, randomly occurring transient outages, adverse weather conditions for the continuity of repair and maintenance activities, and availability of switching operation to isolate failed component from the network configuration are investigated in the reliability algorithm.

The second algorithm models the three-phase distribution network for its operational characteristics. This algorithm provides a detailed three-phase load flow solution for a distribution network. In addition to the three-phase lines, the algorithm also allows consideration of a neutral wire, a ground wire, the mutual coupling between phases and lines, and various type of transformer connections (e.g., wye-wye, wye-delta, etc.) and transformer tap setting. The results from these two algorithms enable the power utilities to improve their service to their customers, i.e., the algorithms allow the power utilities to design, plan, maintain, and operate their existing and future development and expansion of their network configuration which are cost effective and economical.

5.2 Load Point Reliability

This thesis proposes a reliability study of a network whose element (i.e., equipment) failure and renewal processes are characterized by the Weibull statistical distribution. This statistical distribution allows the rate parameters associated with the distribution to be variable as opposed to the classical models (exponential distribution function) which assume a constant value. The use of the exponential distribution to characterize a network's failure, maintenance and renewal processes can be quite problematic and misleading. For example, if the network's electrical equipment is old, often its failure rates may increase with in-service life and be significantly higher than the average failure rate. When average failure rates are used in a reliability model of an old electrical network, the frequency of interruptions can be significantly under-estimated. The converse is also true for modelling relatively new electrical systems whose failure rates are often significantly lower than the average value. A detailed discussion of the significant impact of the use of the Weibull statistical distribution instead of the normally assumed exponential distribution in power system reliability analysis is presented. The reliability techniques used in this research are based on the minimum cut-set approach. A knowledge of the network and individual consumer service reliability levels is an extremely important factor in the design of a utility's and consumer's electrical and electronic systems.

The algorithm developed evaluates the reliability performance of distribution network configuration in terms of customer interruption frequencies and duration. The algorithm is quite general and can be used for reliability evaluation of other parts of power systems (e.g., switching stations). The component active, passive, stuck breaker conditions and common mode failure events is also considered. This permits the inclusion of all the realistic component failure modes in the reliability predictions. The algorithm, in addition, is capable of handling weather dependent failures, unreliability of protective schemes, normally open switches and breakers and component overlapping outages. The output of the algorithm provides a concise and sequential description of various system contingencies that cause interruptions. The computational efficiency of the program comes from the evaluation of only the failure related events. The algorithms for tracing all operational paths to the customers, performing failure modes and effect analysis are quite fast.

The thesis clearly demonstrates the significant difference in the frequency of service interruptions based on the Weibull distribution (i.e., for various shape parameters but the same characteristic life). Reliability results based on the exponential distribution are often expressed as a long term average value independent of the network's electrical equipment in-service life. When the Weibull distribution is used in the reliability model, the frequency of service interruptions is dependent upon the in-service life of the network's equipment. The primary advantage of the use of the Weibull distribution for power system reliability modeling studies is its ability to model the varying rate parameters of a utility's electrical equipment to provide more accurate estimates of service reliability levels during various periods in the life of a utility's electrical system. The flexibility of the Weibull distribution to describe the non-stationary failure and renewal processes of electrical equipment as opposed to the use of exponential distribution provides better estimates of the characteristics of a utility's network performance.

The study revealed the impact of scheduled maintenance activities on a customer's service reliability level. The removal of electrical components from service for maintenance purposes will alter the network's operating configuration which may or may not significantly change the number of operational paths servicing an individual customer location. Changes in the number of operational paths servicing a given customer location is directly correlated with service reliability levels experienced by a customer. In practice, it is important to determine the impact of maintenance activities on the various network operating configurations (e.g., load flow and transient studies).

The operating policies of performing or not performing repair and maintenance activities during adverse weather periods was considered in some detail in this research. It was found that the frequency of interruptions was higher, but not significantly higher, when the repair and maintenance activities were discontinued in adverse weather periods. However, if the extreme elements (e.g., rain) from the adverse weather is high and the components being repaired/maintained are very sensitive to these extreme elements, then continuing repair/maintenance activities during adverse weather condition could cause more harm than good to the system [see Appendix C]. Therefore, depending on the components involved and the severity of the adverse weather condition the decision on repair/maintenance activities can be done. The impact of transient outages on the network configuration was also investigated. The frequency of interruptions due to transient outages is dependent on the component affected. If the component affected happens to reduce the risk of interruptions and its connecting components to a particular customer, then the frequency of interruptions to this particular customer will be lower, while the frequency of interruptions to the other customers may be much higher (i.e., it has the similar effect as removal of components for maintenance purposes).

The inclusion of switching actions in reliability analyses of electrical systems is a vital and important feature. It imposes a three-state model on the system : the state before a fault, the state after a fault but before switching, and the state after switching. The second state has a small average annual outage time because its residence time is dependent on the switching times, although its rate of occurrence can be very large. The importance of switching actions is obvious from the results of Chapter II.

5.3 Unbalanced Three-Phase Load Flow

Distribution networks are characterized by their variable operating configurations which are always in a state of connecting, disconnecting and interconnecting new and old customer loads. Distribution system loads are usually unbalanced due to the single phase loads they serve. The presence of negative-sequence currents due to unbalance at the generator terminals may overheat the rotor of the generator. In addition, because of unpredictable current distributions, protective relays could malfunction causing further interruptions and active power loss could be comparatively higher than expected due to these further interruptions.

The development of a Gauss-Seidel unbalanced three-phase load flow in the phase frame of reference algorithm is described in this thesis. The algorithm provides a detailed load flow solution that includes the mutual coupling terms and the neutral and ground conductors in the load flow methodology to demonstrate several important points that must be considered in the analysis of distribution systems. The results of the unbalanced load flow program are compared with those obtained from a balanced load flow model.

This thesis reveals that even if a network's load are balanced three-phase loads, the phase currents and the power flowing throughout the network can be unbalanced, i.e., significant neutral, ground and earth currents can exist during normal balanced load operation of a power system. This conclusion was based on a load flow model that included the mutual coupling terms between the various phases in the network (i.e., the three-phase conductors, the neutral conductor and the ground conductor). The impact of unbalanced network loads on the phase voltages, line flow and the earth return current are also investigated in the thesis. The more unbalanced the network loads (e.g., single phase network loads) the more severe the unbalance in the line flow and bus voltages, and the higher the earth return currents.

This thesis reveals some disturbing conditions that generally do not surface by modeling systems with the inadequacies of most distribution analysis software [32]. With a significant grounding grid provided by a network, parallel ground paths to earth are developed that somewhat diminish the effect of the voltage-to-neutral displacement. This mitigating property of the distribution system provides an inborn means of reducing voltage unbalance, which can cause serious problems with three-phase motors, although it is not sufficient to eliminate such unbalance. If no methods for identifying such potential unbalanced voltage problem are available (e.g., software which incorporate ground resistance in the analysis) serious shortcomings in providing quality service to the customer can result.

5.4 Future Development

Considerable research is being undertaken to decrease the computational time of reliability and unbalanced load flow algorithms, particularly for a large system (e.g., 1000-10,000 buses). The reliability and the unbalanced load flow of the distribution system caused by system faults (e.g. three-phase fault, line-to-line fault, ground fault, etc.) are also being investigated.

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APPENDIX A

ADDITIONAL PROPERTIES OF WEIBULL DISTRIBUTION

A.1 Statistical Properties

The k-th moment of the Weibull distribution is found as follows :

$$\mu_k' = E(x^k) = \int_0^\infty x^k \frac{\beta}{\alpha} \left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left(-\frac{x}{\alpha}\right)^\beta dx \tag{A.1}$$

Using the transformation :

$$u = \left(\frac{x}{\alpha}\right)^\beta$$

then :

$$du = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} dx$$

and we have :

$$\mu_k' = \alpha^k \int_0^\infty u^{k/\beta} e^{-u} du \tag{A.2}$$

which is readily recognized as a Gamma function and yields :

$$\mu_k' = \alpha^k \Gamma\left(1 + \frac{k}{\beta}\right) \tag{A.3}$$

Hence, the mean of the Weibull distribution is :

$$\mu = \alpha \Gamma\left(1 + \frac{1}{\beta}\right) \tag{A.4}$$

and the variance is :

$$\sigma^2 = \alpha^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right] \tag{A.5}$$

If one assumes the Weibull distribution as a functional form of the failure distribution, the problem of estimating the parameters α and β must be solved. Two methods for solving this problem - the maximum-likelihood and minimum-chi-square methods [49] - are quite involved and for a high degree of accuracy require a high speed computer due to the complexity of the maximum-likelihood and minimum-chi-square functions.

Fortunately, through the use of Weibull probability graph paper, a simple method is available for obtaining the estimates of the parameters α and β of the Weibull distribution.

A.2 *Graphical Estimation of the Parameters*

Graphical estimation of parameters and graphical prediction have great appeal in practice. Basically, in order to use a graphical estimation procedure, a convenient transformation of the cumulative distribution function must be available that changes it into a linear form.

Consider the Weibull cumulative distribution function as follows :

$$F(t) = 1 - \exp\left(-\frac{t}{\alpha}\right)^{\beta} \tag{A.6}$$

which, after it is rearranged and the natural logarithm is taken twice, yields :

$$\ln\left(\ln\frac{1}{1 - F(t)}\right) = \beta \ln t \; - \beta \ln \alpha \tag{A.7}$$

Rearranging this into the standard form for the dependent and the independent variable, gives :

$$\ln t = \frac{1}{\beta} \ln (\ln \frac{1}{1 - F(t)}) + \ln \alpha \qquad \text{(A.8)}$$

which is clearly of the form Y = (1/β) X + A, and will plot as a straight line on rectangular (or semi-logarithmic) (X,Y) graph paper.

Weibull graph paper can be constructed by relabelling the grid lines on the rectangular paper as $Y = \ln t$ and $X = \ln \{ \ln [1/ (1 - F(t))] \}$. In addition, the axes are usually reversed and then β is the slope of the straight-line plot. Figure A.1 shows such a Weibull paper, and Figure A.2 shows the estimated slope, $\underline{\underline{\beta}} = 1.6$. In commercial Weibull paper a printed scale is usually provided for estimating the slope. In Figure A.2, the scale is shown at the top.

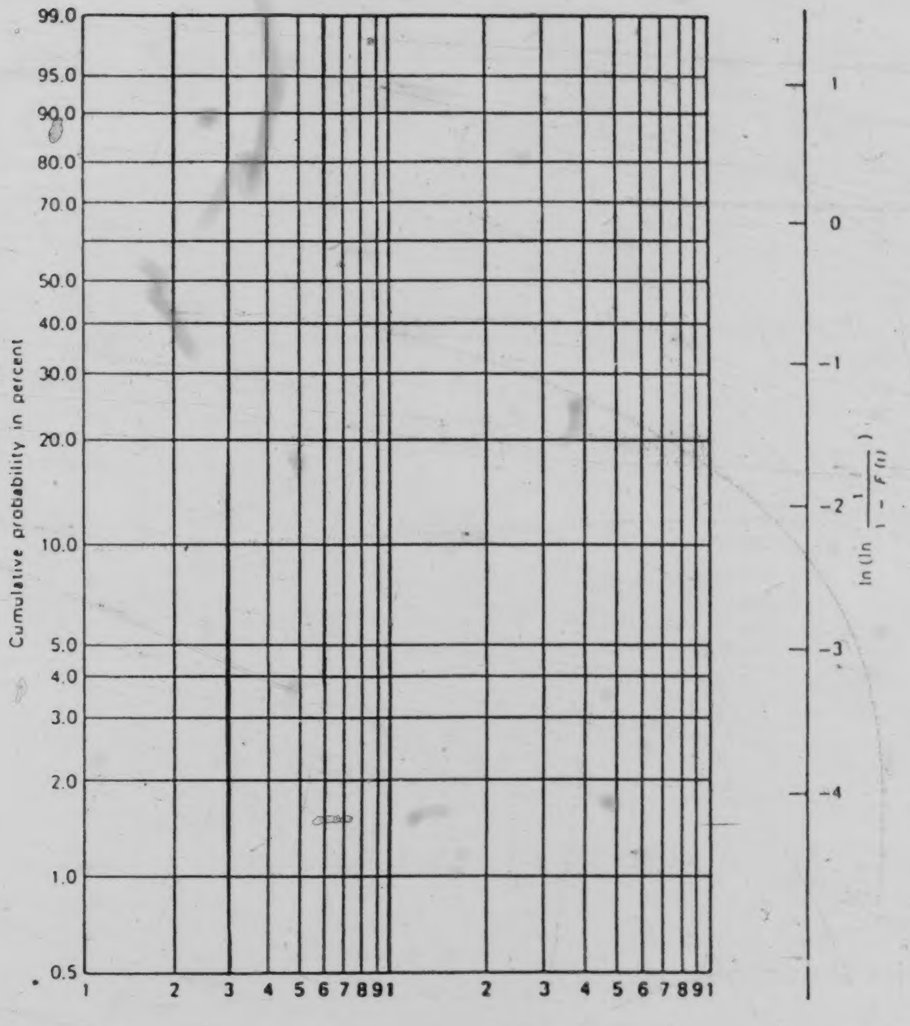


Figure A.1 Weibull probability paper.

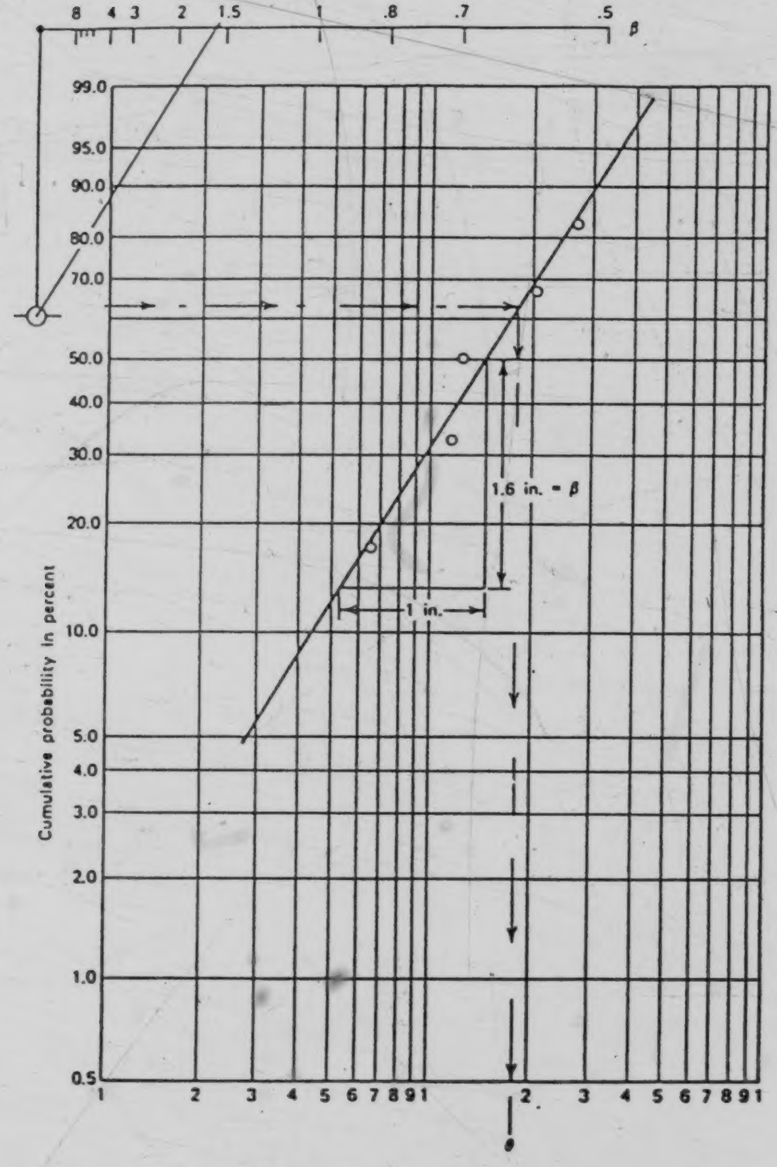


Figure A.2 Graphical parameter estimate for Weibull distribution.

The characteristic life parameter α can be estimated by noting that

$$F(x = \alpha) = 1 - e^{-1} = 0.632.$$

Thus projecting the 63.2% point from the ordinate to the corresponding value on the abscissa gives an estimate of α, $\underline{\underline{\alpha}}$. In Figure A.2, $\underline{\underline{\alpha}}$ = 190 hours.

The mean can also be estimated graphically by substituting $\mu = \alpha \, \Gamma(1+ 1/\beta)$ to yield

$$F(x = \mu) = 1 - \exp \{ - [\Gamma(1 + \frac{1}{\beta})]^{\beta} \} \qquad \text{(A.9)}$$

which is a function of β only. This function is presented in Table A.1. So by locating $F(\mu)$ on the ordinate for a given slope, the mean is found as the corresponding value on the abscissa. In Figure A.2, a slope of 1.6 leads to an estimate mean of 170 hours. The mean computed from the original data is 165.4 hours. Due to inevitable graphical errors, it is best to calculate the mean directly from the original data.

Table A.1 Theoretical relationship between cumulative distribution and shape [18].

SLOPE β	$F(\mu)$
0.5	0.757
1.0	0.632
1.1	0.618
1.2	0.605
1.3	0.594
1.4	0.584
1.5	0.576
1.6	0.568
1.8	0.555
2.0	0.544

A detailed description of how to use the Weibull probability paper to find the parameters can be found in reference [18].

A.3 Examples of Estimating Weibull Parameters

A.3.1 *Maximum-Likelihood Approach*

The maximum-likelihood estimates of scale (α) and shape (β) parameters are obtained by solving

$$n\alpha - \sum_{i=1}^{n} t_i^{\beta} = 0 \tag{A.10}$$

$$\frac{n}{\beta} + \sum_{i=1}^{n} \ln(t_i) - \frac{1}{\alpha}\sum_{i=1}^{n} t_i^{\beta} \ln(t_i) = 0 \tag{A.11}$$

An initial value for β may be obtained by using eqn. (A.10) and

$$E(T) = \alpha^{1/\beta}\, \Gamma\left(1 + \frac{1}{\beta}\right) \tag{A.12}$$

Then find $\bar{\alpha}$ and $\bar{\beta}$ by an iteration procedure. Most investigations of parameter estimation assume β known. If β is known, then

$$\bar{\alpha} = \left(\sum_{i=1}^{r} t_i^{\beta} + (n-r)\, t_r^{\beta} \right) / r \tag{A.13}$$

and $\mathbf{2r}\,\bar{\alpha}\,\mathbf{/a}$ is distributed as χ^2 with 2r degrees of freedom. Thus, the lower 100 γ percent confidence interval on α is given by

$$P\left[\frac{2r\bar{\alpha}}{\alpha} \leq \chi^2_{\gamma,\, 2r} \right] = \gamma \qquad \text{(A.14)}$$

and

$$\bar{R}_L\,(t) = \exp\,(-\,t^{\beta}\,)\,\chi^2_{\gamma,\; 2r/2r\alpha} \qquad \text{(A.15)}$$

A.3.2 *Weibull Paper Approach*

Suppose the 10 ordered observations in a life test are 300, 410, 500, 600, 660, 750, 825, 900, 1050, and 1200 hours. Assume that the Weibull distribution fits the data. The corresponding plotting positions are given by $i/(10+1)$, that is, 0.09, 0.18, 0.27, 0.36, 0.46, 0.55, 0.64, 0.73, 0.83, and 0.91, respectively. Since these points plotted on Weibull probability paper result in a straight line, $\bar{\gamma} = 0$. The line gives $\bar{\beta} = 2.35$ and

$\ln \bar{\alpha} = 4.90$ $\qquad\qquad$ $\bar{\alpha} = 134.3$ (X in hundreds of hours)

A.4 Time Dependent Probability

The relevant state space diagram for a simple single component is shown in Figure A.3. In continuous Markov processes, this diagram is usually represented by the transition rates shown in Figure A.3, that is, by the transition rates $\lambda(t)$ and $\mu(t)$ from the operating and failed states respectively. The transition rates $\lambda(t)$ and $\mu(t)$ are assumed to be constant represented by λ and μ.

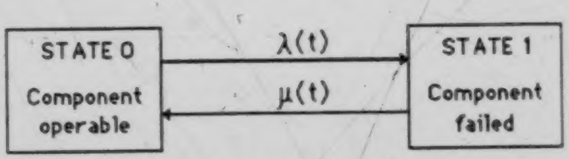


Figure A.3 State space diagram of single component repairable system

Consider an incremental interval of time *dt* which is made sufficiently small so that the probability of two events occurring during this increment of time is negligible. The probability of being in the up state after this time interval *dt* , that is, the probability of being in state 0 of Figure A.3 at time (*t* + *dt*) is

[Probability of being operative at time *t* AND of not failing in time *dt*]

+ [Probability of being failed at time *t* AND of being repaired in time *dt*] .

Let $P_x(t)$ be the probability of failure occurring x times in the interval (0,t), then

$$P_0(t + dt) = P_0(t)\,[\,1 - \lambda\, dt\,] + P_1(t)\,[\,\mu\, dt\,] \qquad (A.16)$$

and

$$P_1(t + dt) = P_1(t)\,[\,1 - \mu\, dt\,] + P_o(t)\,[\,\lambda\, dt\,] \qquad (A.17)$$

From equation (A.16)

$$\frac{P_0(t + dt) - P_0(t)}{dt} = -\lambda P_0(t) + \mu P_1(t)$$

as dt --> 0

$$\left.\frac{P_0(t + dt) - P_0(t)}{dt}\right|_{dt \to 0} = \frac{dP_0(t)}{dt} = \dot{P}_0(t)$$

thus,

$$\dot{P}_0(t) = -\lambda P_0(t) + \mu P_1(t) \qquad (A.18)$$

Similarly, from equation (A.17) $\dot{P}_1(t)$ is :

$$\dot{P}_1(t) = \lambda P_0(t) - \mu P_1(t) \qquad (A.19)$$

The two equations above may be expressed in matrix form as

$$[\,\dot{P}_0(t) \quad \dot{P}_1(t)\,] = [\,P_0(t) \quad P_1(t)\,] \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix} \qquad (A.20)$$

The coefficient matrix in equation (A.20) is not a stochastic transitional probability matrix [10] because the rows of this coefficient matrix summate to zero whereas those of the stochastic transitional probability matrix summate to unity.

Solving equation (A.20) via Laplace transform [10] yields :

$$P_0(t) = \frac{\mu}{\lambda+\mu} + \frac{e^{-(\lambda+\mu)t}}{\lambda+\mu}\,[\,\lambda\,P_0(0)\ -\ \mu\,P_1(0)\,] \tag{A.21}$$

$$P_1(t) = \frac{\lambda}{\lambda+\mu} + \frac{e^{-(\lambda+\mu)t}}{\lambda+\mu}\,[\,\mu\ P_1(0)\ -\ \lambda P_0(0)\,] \tag{A.22}$$

In practice the most likely state in which the system starts is state 0, that is, the system is in an operable condition at zero time. In this case

$$P_0(0) = 0 \qquad \text{and} \qquad P_1(0) = 1$$

Note that $P_0(0) + P_1(0) = 1$ for all initial conditions. Therefore, equations (A.21) and (A.22) reduce to :

$$P_0(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda\,e^{-(\lambda+\mu)}}{\lambda+\mu} \tag{A.23}$$

$$P_1(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda\,e^{-(\lambda+\mu)t}}{\lambda+\mu} \tag{A.24}$$

The probabilities $P_0(t)$ and $P_1(t)$ are the probabilities of the system being in the operating state and failed state respectively as a function of time given that the system started at time $t = 0$ in the operating state.

A.5 Average Reliability Indices

The average reliability indices (e.g., frequency of interruptions, duration of interruptions, etc.) can be approximated as follows :

$$H(T)_{ave} = \frac{1}{N} \sum_{i=1}^{N} H(t_i)_{ave}$$

$$= \frac{1}{N} \left[H(t_1)_{ave} + H(t_2)_{ave} + \ldots + H(t_N)_{ave} \right] \qquad (A.25)$$

where

N is the number of time intervals considered ;

$H(t_i)_{ave}$ is the average value of reliability indices (e.g., frequency of interruptions) for the time interval t_i ;

t_i small time interval where the indices are approximately linear (i.e., approximately constant) , where t_i are of equal size ;

T the given network operating time.

Note that the average reliability indices can also be approximated by the following equation :

$$H(T)_{ave} = \frac{1}{T} \sum_{i=1}^{N} H(t_i)\ D \qquad (A.20)$$

where D is the duration of the time interval where $H(t_i)$ is taken.

APPENDIX B

RESULTS FROM RELIABILITY STUDY - PASSIVE MODE

The passive failure mode reliability indices (i.e., the frequency of interruptions and duration per interruptions at various operating time period) for the 14 bus test network shown in Figure 2.2 for each of the shape parameters of 0.8, 1.0 (exponential distribution), 1.5, 2.0, 2.5, and 3.0 are shown in Figures B.1 to B.10. The average values of the reliability indices are shown in Table B.1.

Table B.1 Average reliability indices experienced by customer element #13

Shape Parameter	Frequency of interruptions (f/yr)	Durations of interruptions (hrs/f)	Total annual duration of interruptions (hrs/yr)
0.8	0.31070	83.9725	34.1046
1.0	0.28112	62.0918	28.4836
1.5	0.07417	4.5733	1.6683
2.0	0.02867	0.6220	0.1280
2.5	0.01096	0.1220	0.0124
3.0	0.00405	0.0254	0.0012

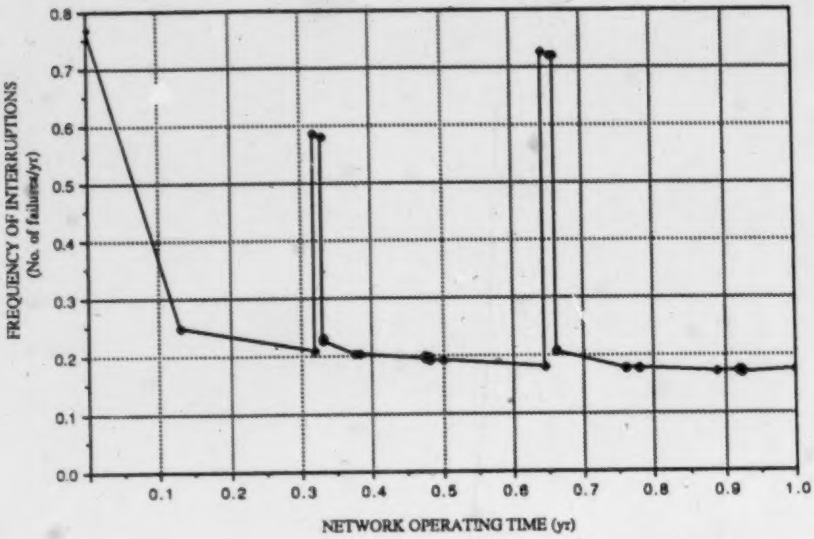


Figure B.1 Frequency of interruptions experienced by customer element #13 at various operating time (shape parameter = 0.8)

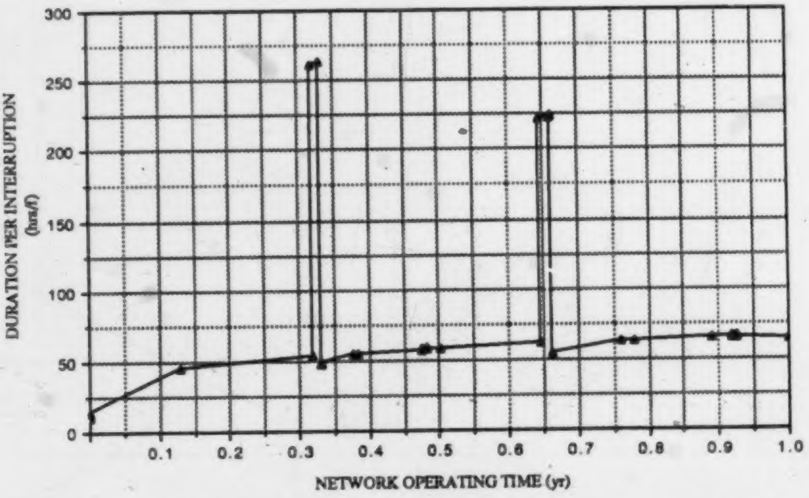


Figure B.2 Duration per interruption experienced by customer element #13 at various operating time (shape parameter = 0.8)

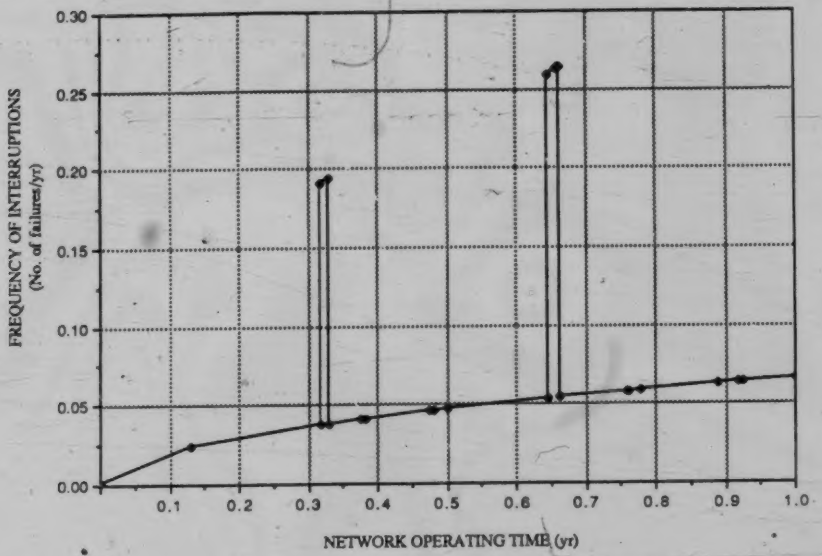


Figure B.3 Frequency of interruptions experienced by customer element #13 at various operating time (shape parameter = 1.5)

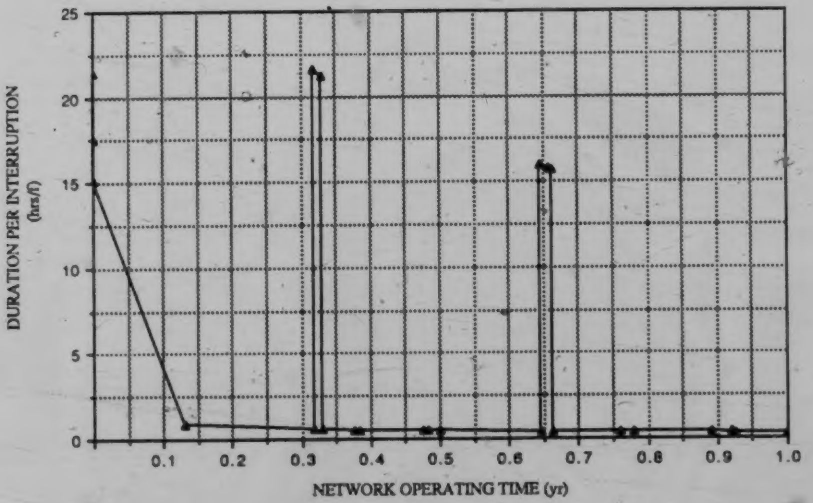


Figure B.4 Duration per interruption experienced by customer element #13 at various operating time (shape parameter = 1.5)

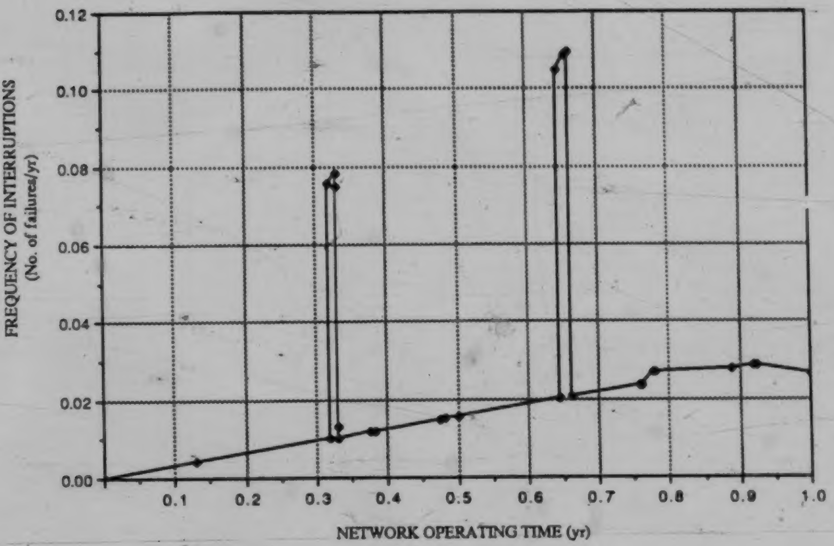


Figure B.5 Frequency of interruptions experienced by element #13 at various operating time (shape parameter = 2.0)

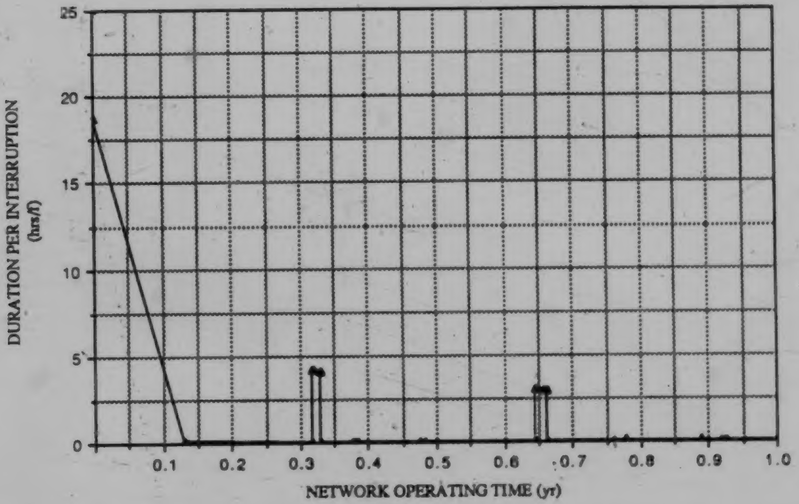


Figure B.6 Duration per interruption experienced by element #13 at various operating time (shape parameter = 2.0)

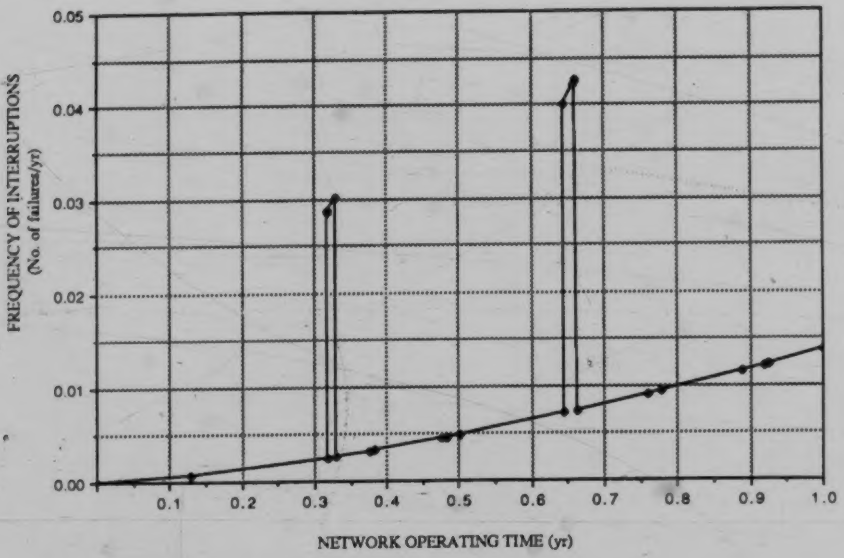


Figure B.7 Frequency of interruptions experienced by element #13 at various operating time (shape parameter = 2.5)

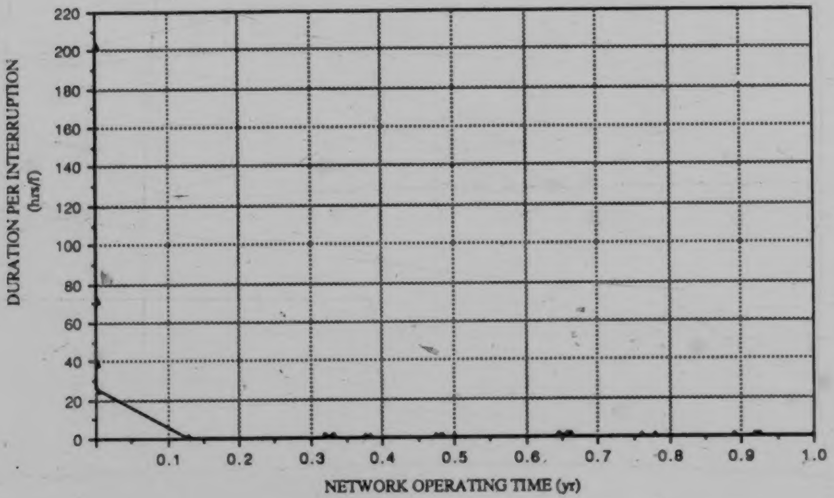


Figure B.8 Duration per interruption experienced by element #13 at various operating time (shape parameter = 2.5)

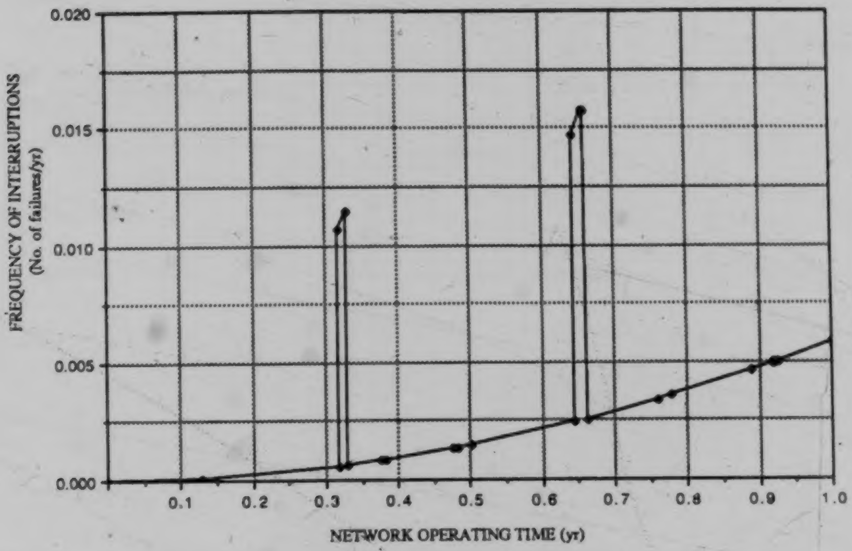


Figure B.9 Frequency of interruptions experienced by element #13 at various operating time (shape parameter = 3.0)

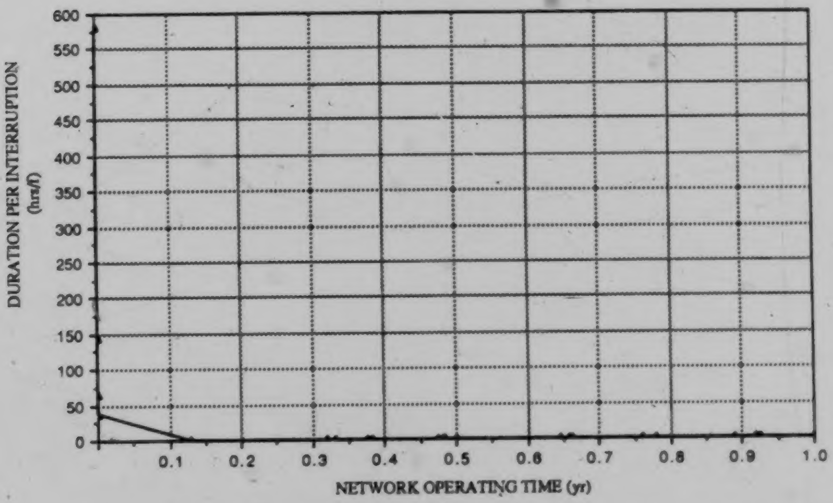


Figure B.10 Duration per interruption experienced by element #13 at various operating time (shape parameter = 3.0)

APPENDIX C

FURTHER RELIABILITY STUDIES

C.1 Impact of Preventive Maintenance

The coordination of preventive maintenance scheduling with reliability analysis can be very effective, especially for a large power system. Figure C.2 illustrates the differences in the frequency of interruptions of a 14 bus test system (Figure C.1) with or without preventive maintenance. It is obvious from the plot that the frequency of interruption is significantly higher when a system does not have preventive maintenance (0.03126611) than when the system with preventive maintenance (0.02670439). Note that all data used in this Appendix are similar to those of Chapter II, unless otherwise stated.

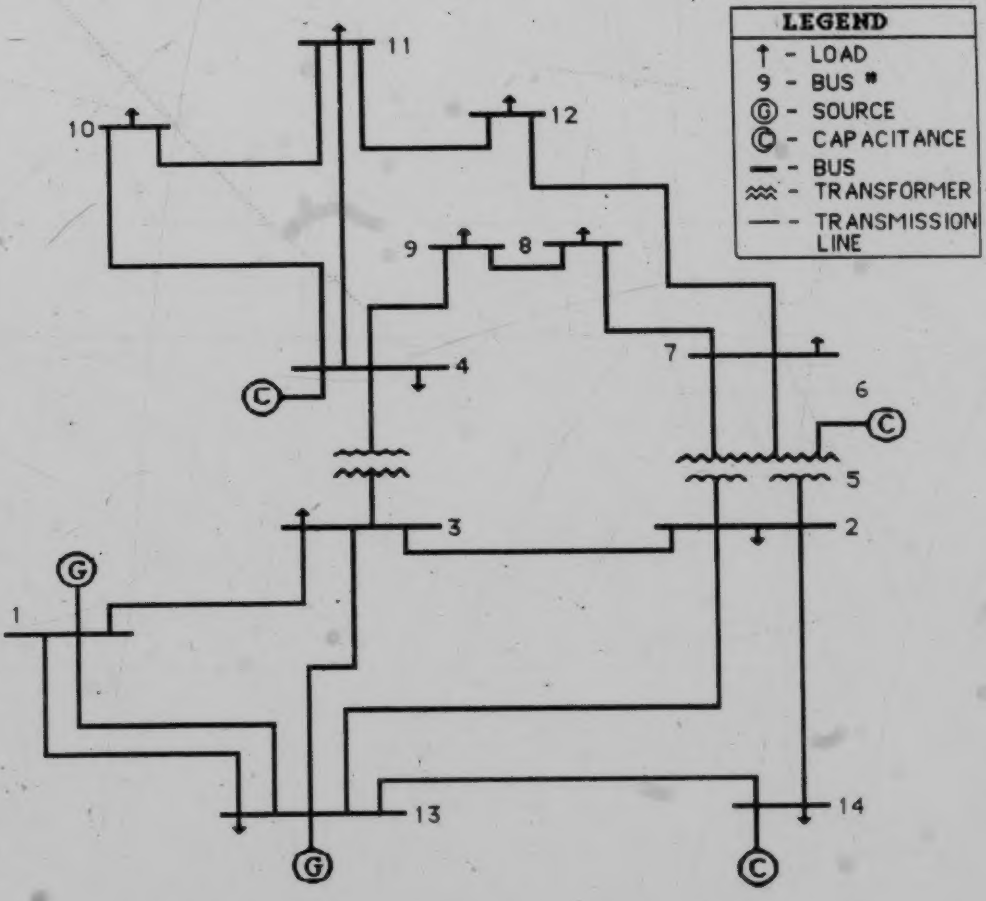

LEGEND
↑ - LOAD
9 - BUS #
Ⓖ - SOURCE
Ⓒ - CAPACITANCE
▬ - BUS
≈ - TRANSFORMER
— - TRANSMISSION LINE
11
10
12
9
8
7
4
C
6
C
5
3
2
G
1
13
G
14
C

Figure C.1a 14 bus test system

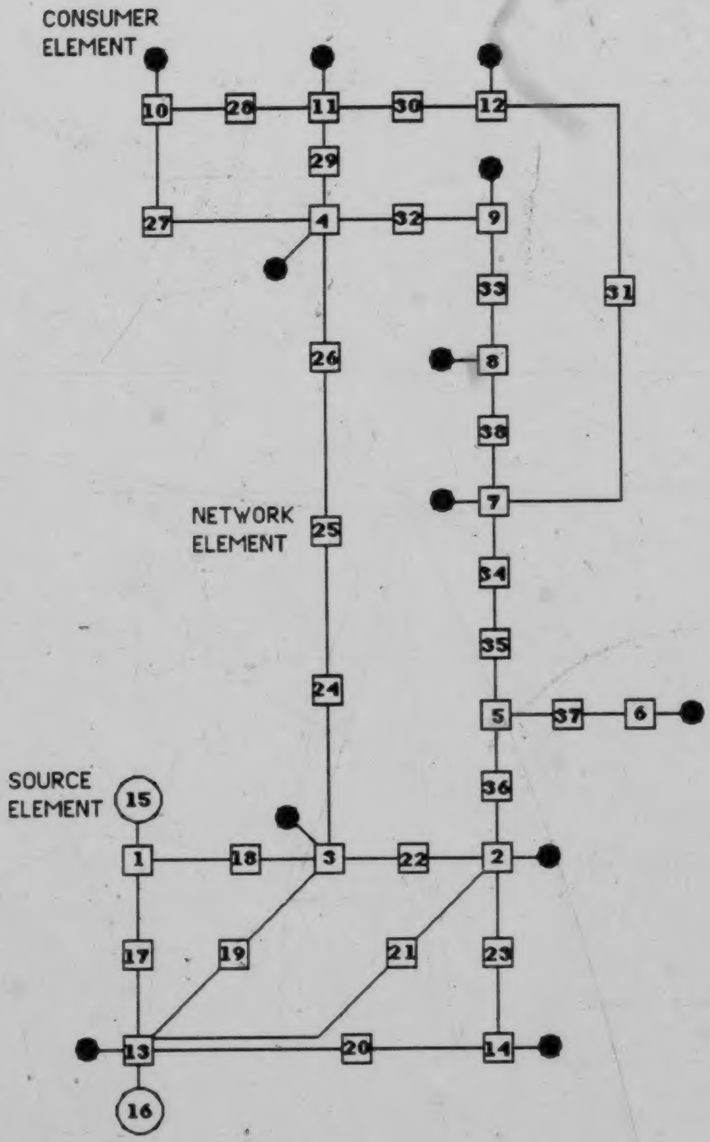

CONSUMER ELEMENT
10
28
11
30
12
29
27
4
32
9
33
31
26
8
38
7
NETWORK ELEMENT
25
34
35
24
5
37
6
SOURCE ELEMENT
15
36
1
18
3
22
2
17
19
21
23
13
20
14
16

Figure C.1b Element block diagram of 14 bus test system

Table C.1 Frequency of interruption at load point #13 with shape parameter of 2.0 - same schedule of preventive maintenance as in Section 2.3

average frequency of interruptions :

with maintenance : ~ 0.021451 f/yr
without maintenance : ~ 0.021455 f/yr

TIME (yr)	with maintenance	without maintenance
0.114155 E-03	0.00000357	0.00000357
0.228310 E-03	0.00000713	0.00000713
0.342465 E-03	0.00001071	0.00001071
0.456620 E-03	0.00001428	0.00001428
0.318607	0.00996162	0.00996162
0.318721	0.00996519	0.00996519
0.474886	0.01484538	0.01484785
0.475000	0.01484894	0.01485142
0.644634	0.02015275	0.02015521
0.644749	0.02015631	0.02015878
0.760045	0.02375630	0.02376367
0.760159	0.02375987	0.02376723
0.918721	0.02871742	0.02872485
0.918835	0.02872099	0.02872842
0.999771	0.03125153	0.03125897
0.999885	0.03125510	0.03126254
0.999999	0.03125867	0.03126611

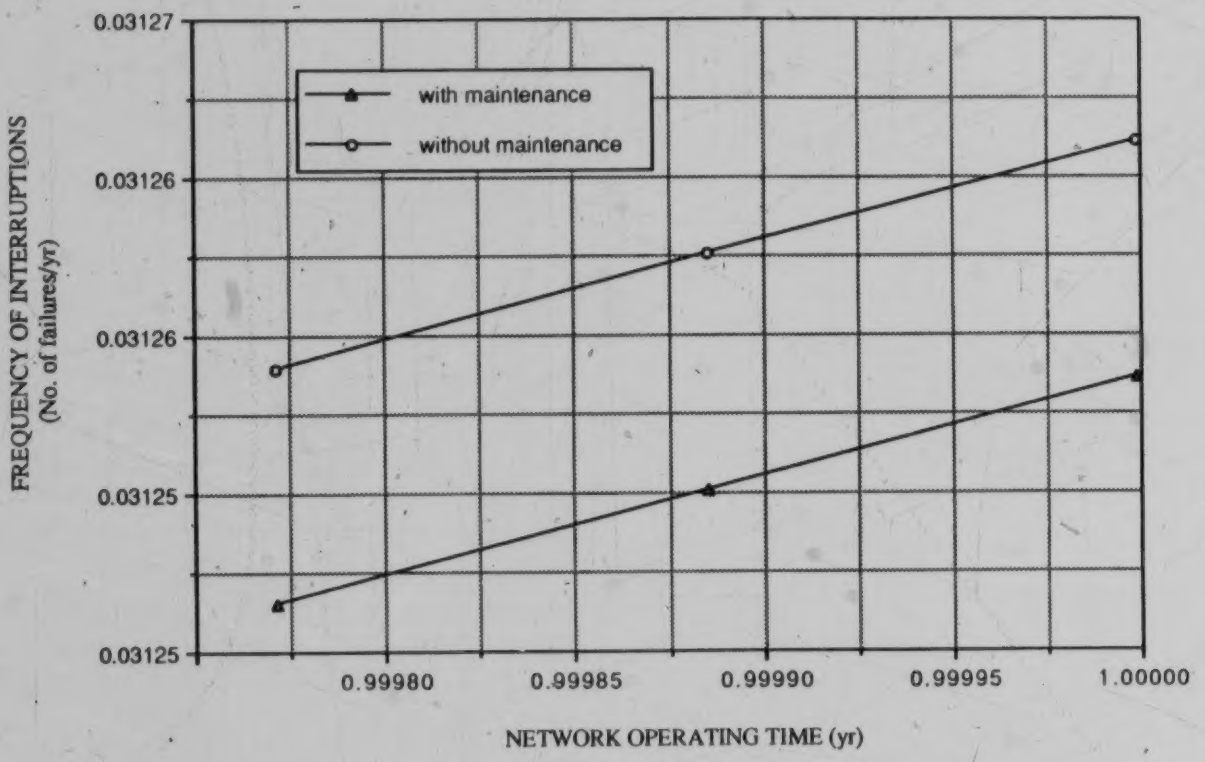


Figure C.2 Frequency of interruptions experienced by customer element #13 of Fig. C.1
(shape parameter = 2.0)

Consider performing preventive maintenance on a small system (Figure C.3) with shape parameter for the failure process of 2.0 and renewal process of 1.0. The reliability indices for operating time from tenth to eleventh year is evaluated and the results are shown in Tables C.2 and C.3. The maintenance activity is scheduled six months later on element #8. Figures C.4 and C.5 illustrate the differences when the small system does not have preventive maintenance (average frequency of interruptions ~ 1.35195 - weather independent, no switching) and when the system with preventive maintenance ((average frequency of interruptions ~ 1.24464 - weather independent, no switching). The average frequency of interruptions for the test system of Figure C.3, with maintenance activities is ~ 0.1651 f/yr, and without maintenance activities is ~ 0.1371 f/yr.

Note that for the case where repair activities are discontinued in adverse weather the frequency of interruptions increases during maintenance period because the remaining components supplying to the load are first order cut elements, i.e., failure of any of these components (in any failure mode) will cut all supply to the load (#10). After the maintenance activities the frequency of interruptions are lower than the frequency of interruptions where maintenance activities are not performed. Hence, depending on the network configuration, preventive maintenance can effectively increase or decrease the reliability of the system during an interval of its life cycle. In general, preventive maintenance is advisable as it improves the reliability of a power system. However, in a remote power distribution area care must be taken in scheduling maintenance so as to minimize its impact on customers.

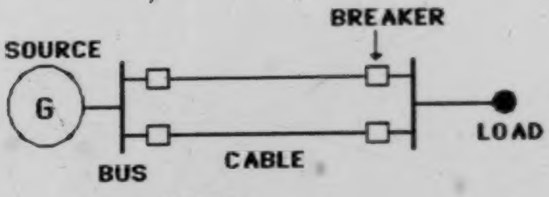

BREAKER
SOURCE
G
LOAD
BUS
CABLE

Figure C.3a Simple test system

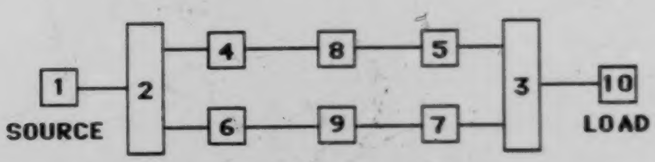

1
SOURCE
2
4
8
5
6
9
7
3
10
LOAD

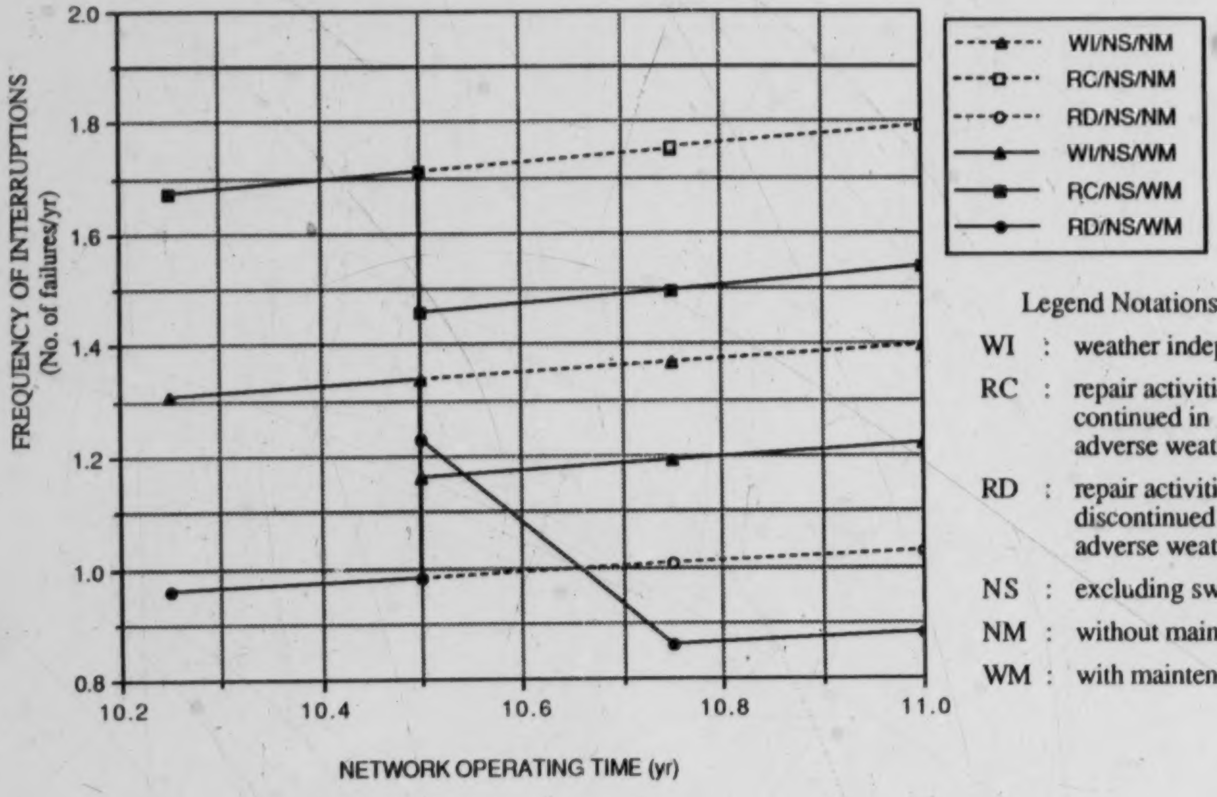
Figure C.3b Element block diagram of the simple test system

Table C.2 Frequency of interruptions experienced by element # 10 - with maintenance

	EXCLUDING SWITCHING			INCLUDING SWITCHING		
Time	weather independent	repair continued	repair discontinued	weather independent	repair continued	repair discontinued
10.25010	1.30790620	1.67346860	0.95811790	1.37830920	1.67356110	1.41206550
10.25020	1.30791850	1.67348580	0.95812798	1.37832260	1.67357830	1.41208080
10.25030	1.30793290	1.67350480	0.95813912	1.37833880	1.67359730	1.41209700
10.25050	1.30794620	1.67352390	0.95814997	1.37835310	1.67361640	1.41211320
10.49980	1.33733460	1.71303460	0.98145503	1.41104790	1.71313190	1.44645980
10.49990	1.33734890	1.71305080	0.98146439	1.41106320	1.71314810	1.44647310
10.50000	1.33735850	1.71306610	0.98147351	1.41107460	1.71316240	1.44648650
10.50010	1.33737180	1.71308140	0.98148239	1.41108890	1.71317770	1.44649980
10.50020	1.33738330	1.71309760	0.98149204	1.41110040	1.71319390	1.44651410
10.50030	1.16246990	1.45912840	1.23199650	1.19648550	1.45912840	1.23199750
10.75010	1.19275950	1.49883650	0.85928512	1.22919940	1.49883840	1.26840700
10.75020	1.19277290	1.49885460	0.85929614	1.22921470	1.49885650	1.26642230
10.75030	1.19278720	1.49887180	0.85930610	1.22922990	1.49887370	1.26843750
10.75050	1.19280050	1.49888990	0.85931641	1.22924420	1.49889180	1.26645280
10.99970	1.22290710	1.53845120	0.88261014	1.26184560	1.53845500	1.30078220
10.99980	1.22292140	1.53846840	0.88262093	1.26186080	1.53847310	1.30079840
10.99990	1.22293470	1.53848740	0.88263196	1.26187520	1.53849120	1.30081460
11.00000	1.22294810	1.53850560	0.88264275	1.26189040	1.53850940	1.30082990

Table C.3 Frequency of interruptions experienced by element # 10 - without maintenance

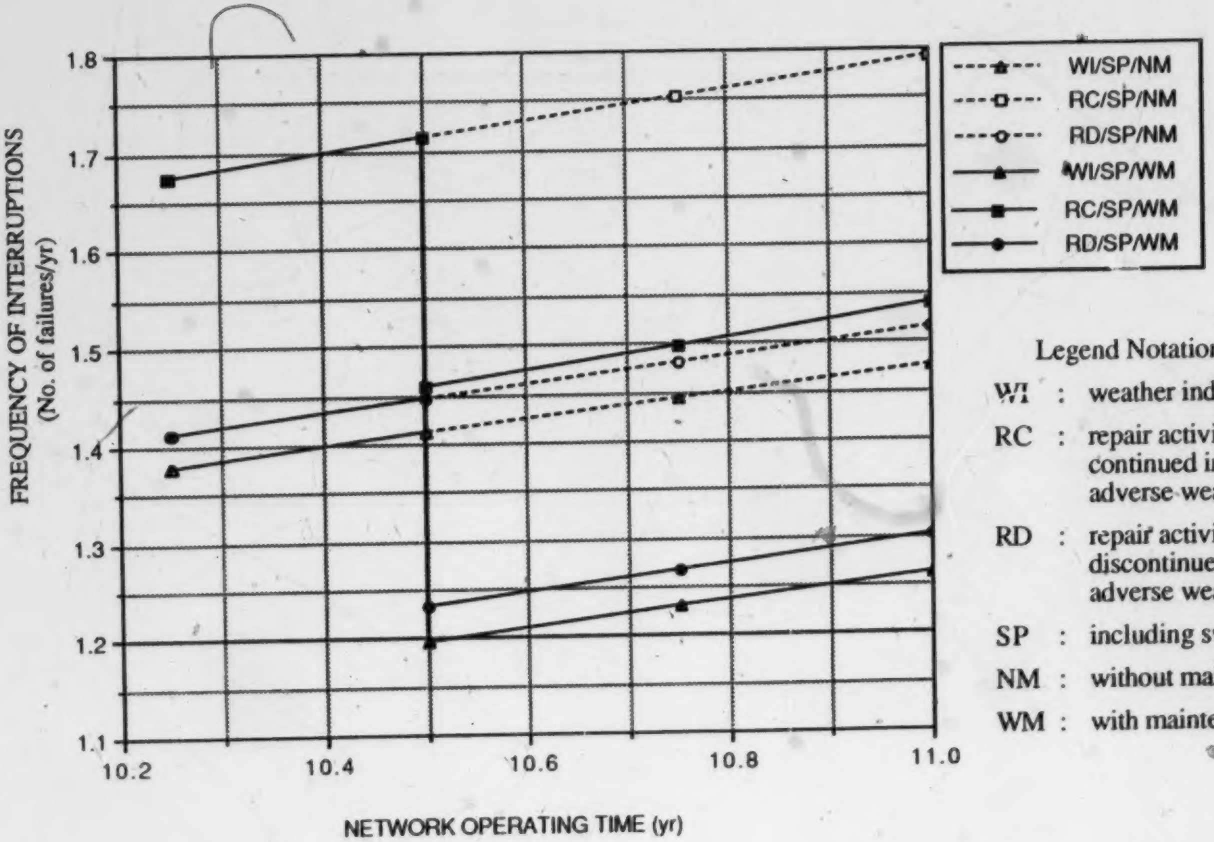
	EXCLUDING SWITCHING			INCLUDING SWITCHING		
Time	weather independent	repair continued	repair discontinued	weather independent	repair continued	repair discontinued
10.25010	1.30790620	1.67346860	0.95811790	1.37830920	1.67356110	1.72280690
10.25020	1.30791850	1.67348580	0.95812798	1.37832260	1.67357830	1.72282510
10.25030	1.30793290	1.67350480	0.95813912	1.37833880	1.67359730	1.72284510
10.25050	1.30794620	1.67352390	0.95814997	1.37835310	1.67361640	1.72286510
10.49980	1.33733460	1.71303460	0.98145503	1.41104790	1.71313190	1.76476960
10.49990	1.33734890	1.71305080	0.98146439	1.41106320	1.71314810	1.76478670
10.50000	1.33736040	1.71306900	0.98147506	1.41107650	1.71316620	1.76480580
10.50010	1.33737560	1.71308710	0.98148566	1.41109370	1.71318340	1.76482490
10.50020	1.33738800	1.71310620	0.98149663	1.41110710	1.71320250	1.76484490
10.50030	1.33740140	1.71312240	0.98150688	1.41112230	1.71321960	1.76486300
10.75010	1.36673360	1.75265220	1.00485420	1.44383530	1.75275420	1.80684570
10.75020	1.36674690	1.75267030	1.00486470	1.44384960	1.75277230	1.80686470
10.75030	1.36676030	1.75268650	1.00487520	1.44386480	1.75278850	1.80688190
10.75050	1.36677360	1.75270560	1.00488570	1.44388010	1.75280670	1.80690190
10.99970	1.39593600	1.79208850	1.02818010	1.47648240	1.79219440	1.84878730
10.99980	1.39594840	1.79210660	1.02819060	1.47649670	1.79221250	1.84880640
10.99990	1.39596180	1.79212570	1.02820110	1.47651200	1.79223160	1.84882640
11.00000	1.39597510	1.79214290	1.02821250	1.47652630	1.79224970	1.84884550



Legend Notations

WI : weather independent
RC : repair activities continued in adverse weather
RD : repair activities discontinued in adverse weather
NS : excluding switching
NM : without maintenance
WM : with maintenance

Figure C.4 Frequency of interruptions experienced by element # 10 - excluding switching



Legend Notations

WI : weather independent

RC : repair activities continued in adverse weather

RD : repair activities discontinued in adverse weather

SP : including switching

NM : without maintenance

WM : with maintenance

Figure C.5 Frequency of interruptions experienced by element # 10 - including switching

C.2 Impact of Duration of A Maintenance

The impact of duration of a maintenance on the reliability of a system is investigated, i.e., what is the effect when a preventive maintenance takes longer or shorter time than anticipated. Maintenance activities are performed at the end of six months (0.5 year) for various duration. The respective average frequency of interruptions for each of the study at t = 0.751142 yr are compared as shown in Table C.4. Figure C.6 and Figure C.7 show the impacts for a small system (Figure C.3) and a large system (Figure C.1).

From Figure C.6 and Table C.4, it can be seen that if the scheduled duration of maintenance for a small system is long (greater than 8.0 hours), the average frequency of interruptions decreases to a steady state values (~ 0.164628), that is to say for a small system exceeding the anticipated duration of a maintenance is not critical if the scheduled duration is long. However, if the scheduled duration is short (~ 4.0 hours) then exceeding the duration of a maintenance by an hour or two would significantly increases the frequency of interruptions to a customer. Note that the frequency of interruptions is significantly higher when the duration of a maintenance is approximately 2.0 hours. This is because from the point of view at t = 0.751142 yr the maintained components have been exposed to more stress as compared to the other durations since it is assumed that after maintained components are as good as new, i.e., very low failure rate or 100% reliable. Hence, it can be said that for a small system it might be better to have occasional maintenance of long duration (preferably ~ 4.0 hours) than to have regular maintenance of short duration (2.0 hours or less).

Table C.4 Average frequency of interruptions at t = 0.751142 yr, with element #8 being maintained for various time intervals.

DURATION (hrs)	FREQUENCY (f/yr)
2.0	0.16472859
4.0	0.16460345
8.0	0.16462863
12.0	0.16462861
24.0	0.16462858
48.0	0.16462850

Similarly, from Figure C.7 and Table C.5, it can be seen that if the scheduled duration of maintenance for a large system is greater than 12.0 hours, the frequency of interruptions decreases to a steady state values (~ 0.0285549), that is to say for a large system exceeding the anticipated duration of a maintenance is not critical if the scheduled duration is long. However, if the scheduled duration is short (4.0 hours or less) then exceeding the duration of a maintenance by an hour or two would significantly increase the frequency of interruptions to a customer. Hence, it can be said that for a large system it is better to have regular maintenance of short duration than to have occasional maintenance of long duration.

Table C.5 Average frequency of interruptions at t = 0.913356 yr, with element #16 being maintained for various time intervals.

DURATION (hrs)	FREQUENCY (f/yr)
2.0	0.02855140
4.0	0.02855143
8.0	0.02855500
12.0	0.02855499
24.0	0.02855498
48.0	0.02855496

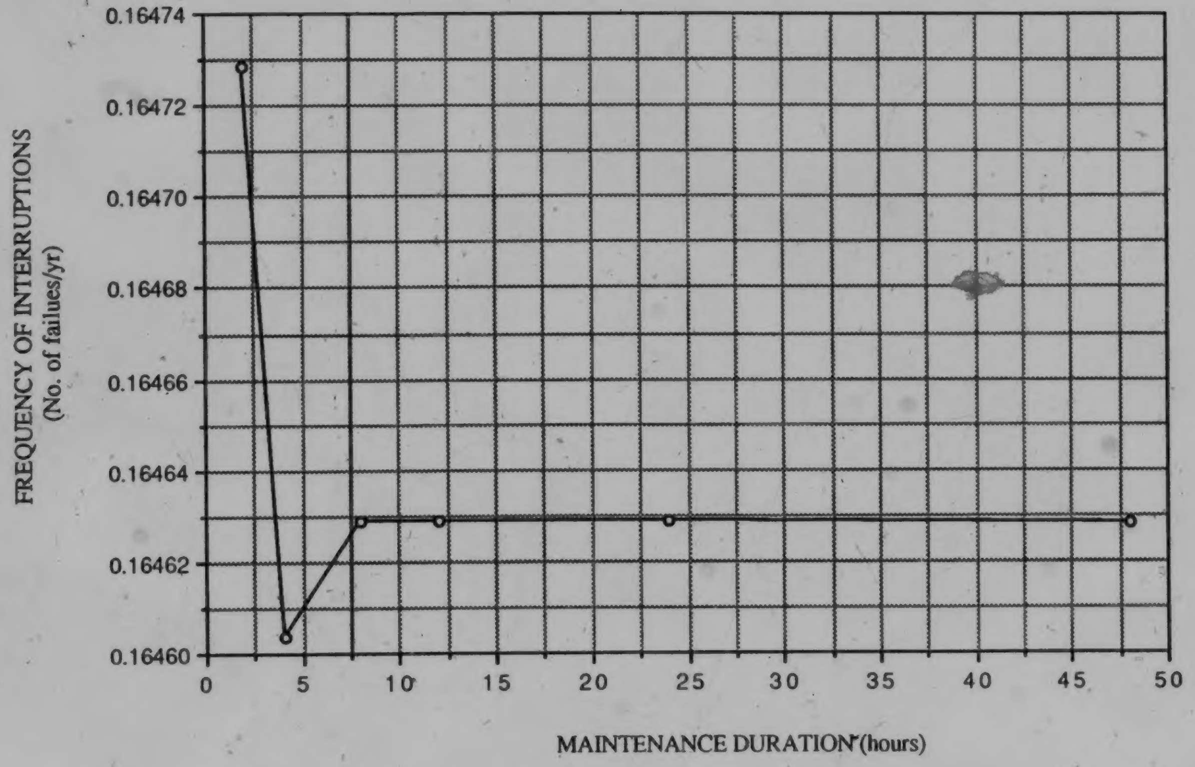


Figure C.6 Average frequency of interruptions experienced by elelemnt #10 of Fig. C.3
- with element #8 being maintained (shape parameter = 2.0)
- at time t = 0.751142 yr

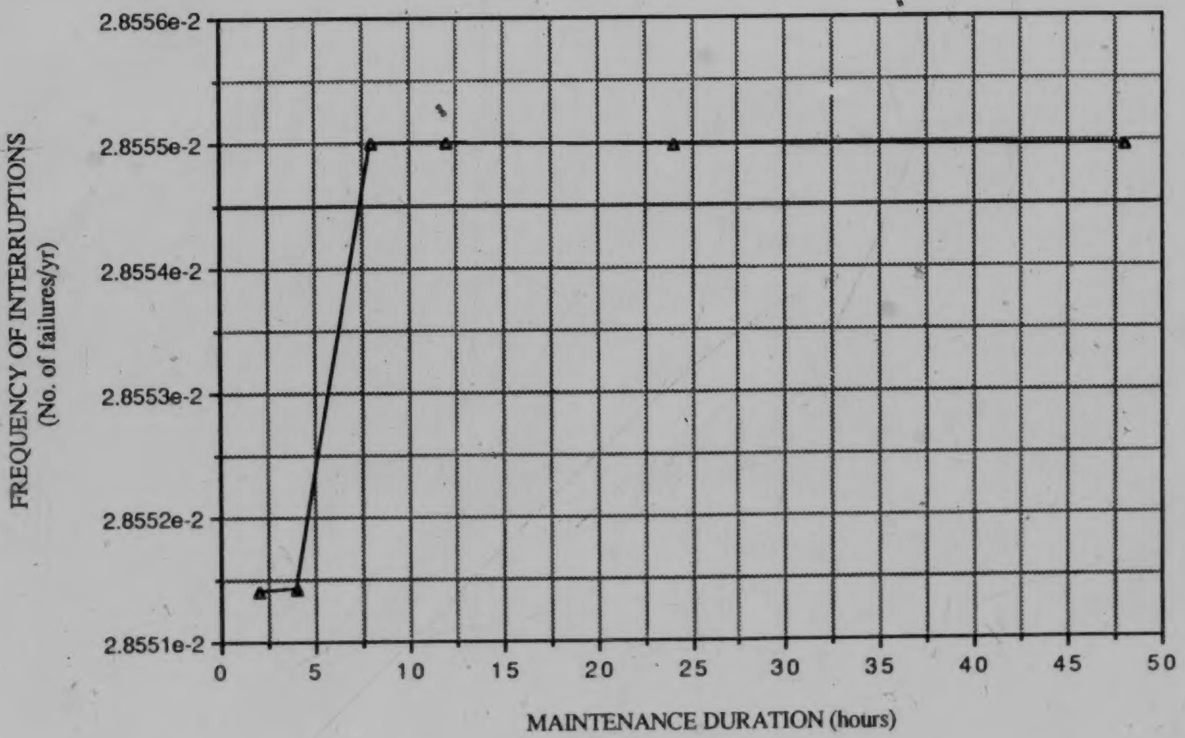


Figure C.7 Average frequency of interruptions experienced by element #13 of Fig. C.1
- with element #16 being maintained (shape parameter = 2.0)
- at $t = 0.913356$ yr.

C.3 Impact of Component Aging

The reliability indices of Figure C.3 were evaluated over a 50 years period, and the results are tabulated in Table C.6. The failure process is assumed to be of Rayleigh distribution (i.e., Weibull distribution with parameters of 2.0) and the renewal process is assumed to be of Exponential distribution (i.e., Weibull distribution with parameters of 1.0). Figures C.8 and C.9 illustrate the characteristic of the test system in Figure C.3.

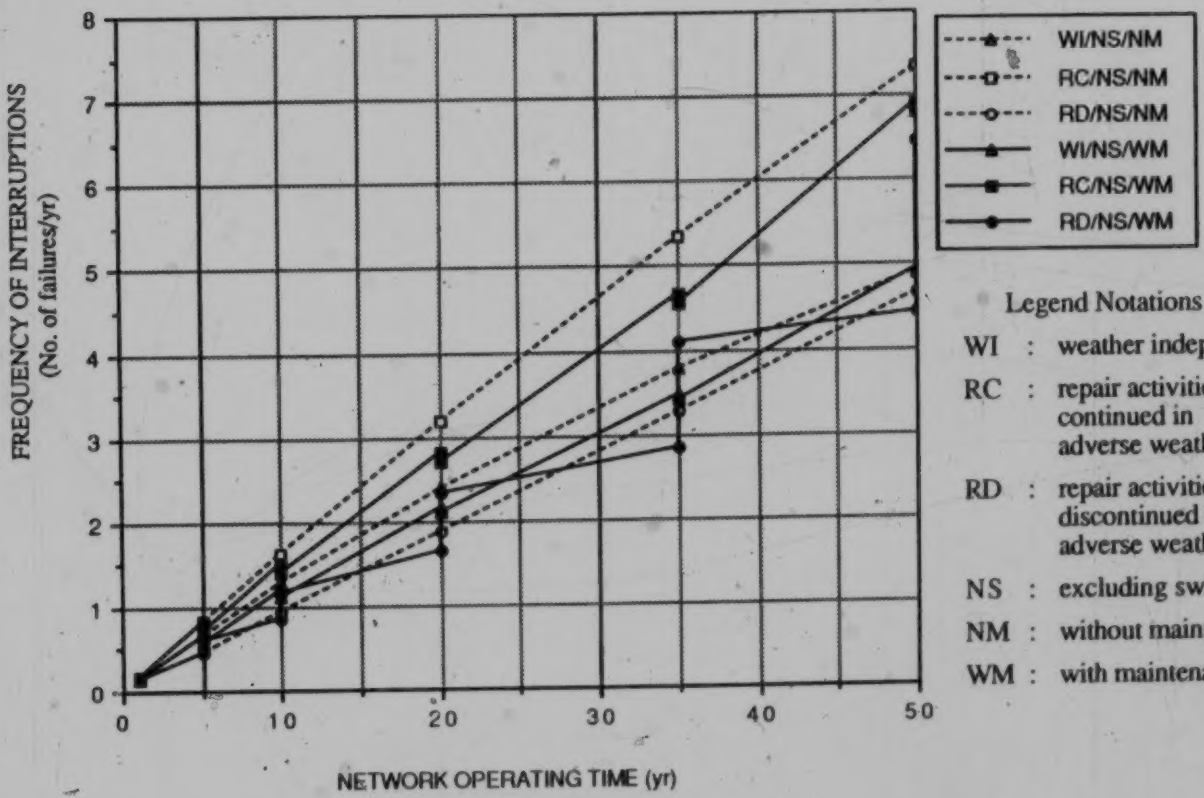
From Figure C.8, it is clear that the frequency of interruptions increases linearly with time at a rate of approximately 0.07 f/yr. Note that the maintenance activities are scheduled to be performed at every five years (i.e., at 5, 10, 15 etc.). It can be seen from Table C.6 and Figures C.8 and C.9 that network with regular maintenance activities reduces the frequency of interruptions.

Table C.6a Frequency of interruptions experienced by element # 10 over 50 years
- with maintenance on element #8 at every 5 years

	EXCLUDING	SWITCHING		INCLUDING	SWITCHING	
Time	weather independent	repair continued	repair discontinued	weather independent	reapir continued	repair discontinued
0.99988598	0.13674337	0.16757572	0.14454782	0.12094700	0.16757667	0.14968556
1.00000000	0.13675880	0.16759485	0.14455992	0.12095743	0.16759580	0.14970016
4.99989030	0.66324306	0.82845116	0.46738756	0.33840257	0.82847387	0.68878937
5.00000000	0.66325551	0.82846713	0.46739662	0.33840889	0.82848984	0.68880272
5.00010970	0.66326791	0.82848299	0.46740568	0.33841527	0.82850569	0.68881619
5.00022980	0.66328031	0.82849884	0.46741486	0.33842146	0.82852155	0.68882954
5.00034050	0.57057184	0.70566618	0.58668792	0.57057184	0.70566618	0.58668828
5.00045970	0.57058626	0.70568430	0.58670282	0.57058626	0.70568430	0.58670348
9.99989030	1.19517610	1.51263620	0.86540681	0.61306751	1.51267720	1.27542880
10.00000000	1.19518850	1.51265140	0.86541623	0.61307347	1.51269250	1.27544210
10.00010000	1.19520090	1.51266770	0.86542541	0.61307991	1.51270870	1.27545640
10.00020000	1.19521240	1.51268290	0.86543459	0.61308587	1.51272390	1.27546980
10.00030000	1.11011980	1.39158820	1.17333220	1.11011980	1.39158820	1.17333320
10.00050000	1.11013510	1.39160630	1.17334650	1.11013510	1.39160630	1.17334840
19.99989300	2.17731860	2.82529160	1.66150670	1.13048080	2.82536600	2.44871620
20.00000000	2.17733000	2.82530690	1.66151620	1.13048740	2.82538130	2.44873140
20.00010700	2.17733860	2.82532020	1.66152480	1.13049220	2.82539370	2.44874100
20.00019800	2.10584550	2.70754720	2.34660340	2.10584550	2.70754720	2.34660430
20.00030500	2.10585880	2.70756440	2.34661770	2.10585880	2.70756440	2.34662060
34.99989300	3.47535320	4.66588310	2.85566710	1.83399770	4.66600130	4.20865820
35.00000000	3.47536470	4.66590120	2.85567860	1.83400440	4.66601940	4.20867540
35.00010700	3.47537230	4.66591170	2.85568520	1.83400820	4.66602900	4.20868590
35.00019800	3.42094800	4.55278780	4.10654260	3.42094800	4.55278780	4.10654540
35.00030500	3.42095570	4.55279920	4.10655210	3.42095570	4.55279920	4.10655690
49.99989300	4.94296460	6.90705200	4.44788070	2.65771770	6.90721800	6.55524440
50.00000000	4.94297310	6.90706440	4.44789120	2.65772250	6.90722940	6.55525680
50.00010700	4.94297890	6.90707490	4.44789510	2.65772530	6.90724090	6.55527110
50.00019800	4.90661330	6.79961300	6.45312500	4.90661330	6.79961300	6.45312880
50.00030500	4.90662380	6.79962640	6.45313450	4.90662380	6.79962640	6.45314120

Table C.6b Frequency of interruptions experienced by element # 10 over 50 years - without maintenance

	EXCLUDING	SWITCHING		INCLUDING	SWITCHING	
Time	weather independent	repair continued	repair discontinued	weather independent	repair continued	repair discontinued
0.99988598	0.13674337	0.16757572	0.14454782	0.12094700	0.16757667	0.14968556
1.00000000	0.13675880	0.16759485	0.14455992	0.12095743	0.16759580	0.14970016
4.99989030	0.66324306	0.82845116	0.46738756	0.33840257	0.82847387	0.68878937
5.00000000	0.66325766	0.82847005	0.46739829	0.33841014	0.82849276	0.68880510
5.00010970	0.66327220	0.82848883	0.46740896	0.33841759	0.82851142	0.68882084
5.00022980	0.66328669	0.82850742	0.46741956	0.33842504	0.82853007	0.68883657
5.00034050	0.66330141	0.82852602	0.46743053	0.33843237	0.82854873	0.68885249
5.00045970	0.66331530	0.82854402	0.46744078	0.33843964	0.82856667	0.68886763
9.99989030	1.27830220	1.63375660	0.93472725	0.66346699	1.63384440	1.37759210
10.00000000	1.27831550	1.63377480	0.93473786	0.66347456	1.63386250	1.37760830
10.00010000	1.27832980	1.63379290	0.93473749	0.66348201	1.63388160	1.37762450
10.00020000	1.27834220	1.63381100	0.93475974	0.66348892	1.63389970	1.37763980
10.00030000	1.27835660	1.63382910	0.93477035	0.66349632	1.63391780	1.37765600
10.00050000	1.27836990	1.63384720	0.93478090	0.66350365	1.63393590	1.37767120
19.99989300	2.38228030	3.17865750	1.86946300	1.27972890	3.17899130	2.75520040
20.00000000	2.38229370	3.17867760	1.86947540	1.27973650	3.17901130	2.75521760
20.00010700	2.38230420	3.17869470	1.86948590	1.27974320	3.17902850	2.75523280
20.00019800	2.38231560	3.17871090	1.86949630	1.27974890	3.17904470	2.75524810
20.00030500	2.38232520	3.17872710	1.86950590	1.27975460	3.17906090	2.75526330
34.99989300	3.77808190	5.34462930	3.27157690	2.12719540	5.34557340	4.82162190
35.00000000	3.77809140	5.34464450	3.27158740	2.12720110	5.34558870	4.82163620
35.00010700	3.77809720	5.34465600	3.27159500	2.12720580	5.34560110	4.82164760
35.00019800	3.77810760	5.34467120	3.27160450	2.12721160	5.34561540	4.82166290
35.00030500	3.77811910	5.34468650	3.27161500	2.12721820	5.34563060	4.82167910
49.99989300	4.92884350	7.34719750	4.67368790	2.89542770	7.34898190	6.88803960
50.00000000	4.92885020	7.34721090	4.67369650	2.89543250	7.34899520	6.88805390
50.00010700	4.92885680	7.34722230	4.67370610	2.89543630	7.34900670	6.88806530
50.00019800	4.92886350	7.34723380	4.67371460	2.89544110	7.34901810	6.88807960
50.00030500	4.92887400	7.34725480	4.67372990	2.89544770	7.34903910	6.88809970



Legend Notations

WI : weather independent

RC : repair activities continued in adverse weather

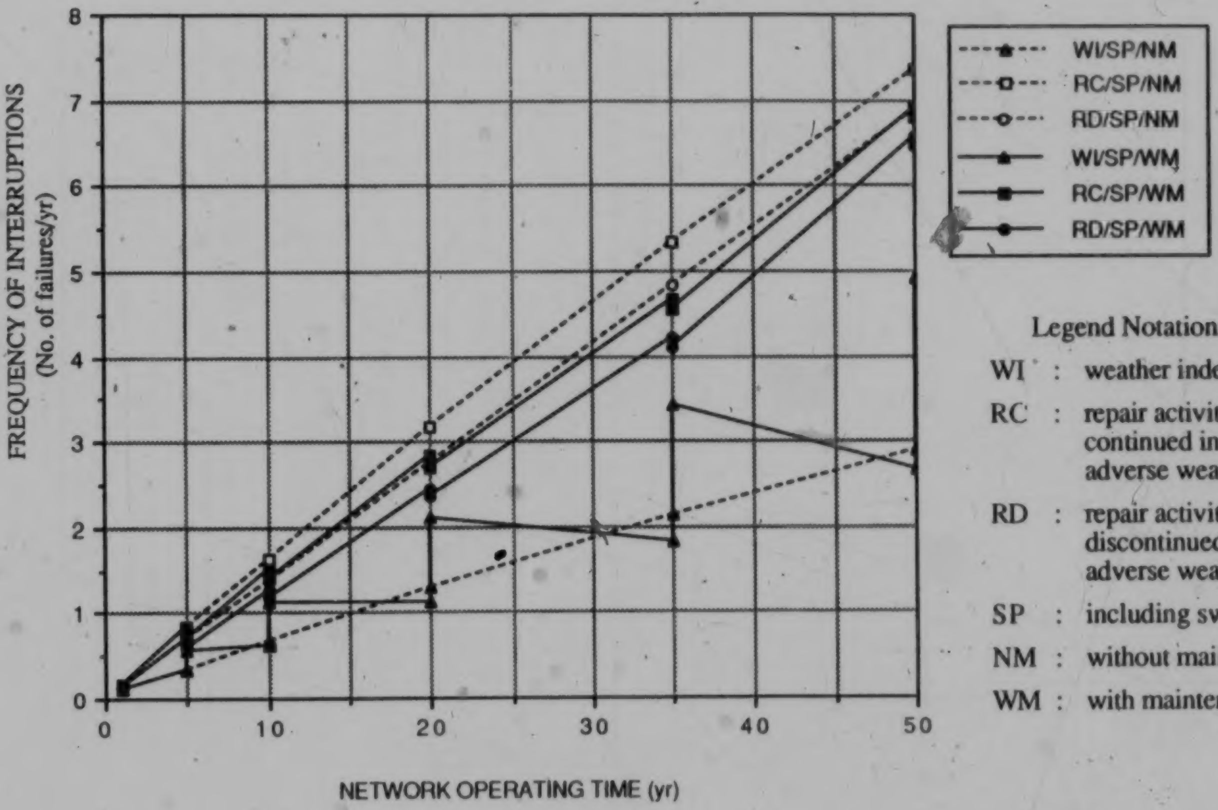
RD : repair activities discontinued in adverse weather

NS : excluding switching

NM : without maintenance

WM : with maintenance

Figure C.8 Frequency of interruptions experienced by element # 10 - excluding switching



Legend Notations

WI : weather independent

RC : repair activities continued in adverse weather

RD : repair activities discontinued in adverse weather

SP : including switching

NM : without maintenance

WM : with maintenance

Figure C.9 Frequency of interruptions experienced by element # 10 - including switching

C.4 Analysis Over Two Months Period With Shape Parameters of 2.0

A more detail analysis of the reliability indices for the network configuration of Figure C.3, over a two months period is illustrated in this section. Using the shape parameter of 2.0, the average frequency of interruptions experienced by the load point at every hour in a two months period is evaluated. The results are summarized as shown in Table C.7 and Figures C.10 to C.11. It is obvious from the results that there is a steady increase in the frequency of interruptions as time increases. The average frequency of interruptions over the two months period for the weather independent, excluding and including switching actions, case study are ~ 0.002259 and ~ 0.002254, respectively. The average frequency of interruptions over the two months period when repair activities are continued in adverse weather, excluding and including switching actions, case study are ~ 0.002755 and ~ 0.002750, respectively ; while the average frequency of interruptions when repair activities are discontinued in adverse weather, excluding and including switching actions, case study are ~ 0.002760 and ~ 0.002755, respectively.

As discussed in Chapter II the weather dependent frequency of interruptions are higher when the repair and maintenance activities are discontinued in adverse weather, this could be due to the exposition of the operating components to higher stress level during the repair and maintenance activities.

Table C.7 Reliability indices of Figure 2.12 with shape parameter of 2.0 over two months period

TIME (YEAR)		EXCLUDING SWITCHING			INCLUDING SWITCHING	
	WEATHER INDEPENDENT	REPAIR CONTINUED	REPAIR DISCONTINUED	WEATHER INDEPENDENT	REPAIR CONTINUED	REPAIR DISCONTINUED
0.00011416	0.00000302	0.00000368	0.00000369	0.00000301	0.00000367	0.00000368
0.00022831	0.00000604	0.00000736	0.00000738	0.00000602	0.00000735	0.00000736
0.00034247	0.00000905	0.00001104	0.00001107	0.00000904	0.00001102	0.00001104
0.01906400	0.00050396	0.00061463	0.00061604	0.00050302	0.0006.347	0.00061467
0.01917810	0.00050698	0.00061831	0.00061973	0.00050603	0.00061714	0.00061835
0.01929230	0.00050999	0.00062199	0.00062342	0.00050904	0.00062081	0.00062203
0.03824210	0.00101094	0.00123300	0.00123580	0.00100905	0.00123065	0.00123304
0.03835620	0.00101395	0.00123669	0.00123949	0.00101206	0.00123432	0.00123672
0.03847040	0.00101697	0.00124037	0.00124318	0.00101507	0.00123800	0.00124040
0.05742010	0.00151791	0.00185144	0.00185559	0.00151508	0.00184786	0.00185144
0.05753420	0.00152093	0.00185512	0.00185928	0.00151809	0.00185154	0.00185512
0.05764840	0.00152395	0.00185880	0.00186297	0.00152110	0.00185521	0.00185880
0.07659823	0.00202489	0.00246993	0.00247540	0.00202111	0.00246511	0.00246987
0.07671231	0.00202791	0.00247361	0.00247909	0.00202412	0.00246878	0.00247355
0.07682651	0.00203092	0.00247729	0.00248278	0.00202713	0.00247246	0.00247723
0.08481729	0.00224217	0.00273501	0.00274104	0.00223798	0.00272966	0.00273492
0.08493149	0.00224518	0.00273869	0.00274473	0.00224099	0.00273333	0.00273860
0.08504558	0.00224820	0.00274237	0.00274842	0.00224400	0.00273700	0.00274228
0.09577632	0.00253187	0.00308846	0.00309524	0.00252714	0.00308239	0.00308832
0.09589040	0.00253488	0.00309215	0.00309893	0.00253015	0.00308607	0.00309201
0.09600449	0.00253790	0.00309583	0.00310262	0.00253317	0.00308974	0.00309569

Table C.7 - continue Reliability indices of Figure 2.12 with shape parameter of 2.0 over two months period

TIME (YEAR)		EXCLUDING SWITCHING			INCLUDING SWITCHING	
	WEATHER INDEPENDENT	REPAIR CONTINUED	REPAIR DISCONTINUED	WEATHER INDEPENDENT	REPAIR CONTINUED	REPAIR DISCONTINUED
0.11495399	0.00303884	0.00370704	0.00371510	0.00303317	0.00369970	0.00370680
0.11506802	0.00304185	0.00371072	0.00371879	0.00303618	0.00370337	0.00371048
0.11518198	0.00304487	0.00371441	0.00372248	0.00303919	0.00370705	0.00371416
0.13413203	0.00354581	0.00432567	0.00433498	0.00353920	0.00431703	0.00432530
0.13424599	0.00354883	0.00432935	0.00433867	0.00354221	0.00432071	0.00432898
0.13436002	0.00355185	0.00433303	0.00434236	0.00354522	0.00432438	0.00433267
0.15331000	0.00405279	0.00494433	0.00495489	0.00404523	0.00493439	0.00494383
0.15342402	0.00405581	0.00494801	0.00495858	0.00404824	0.00493806	0.00494751
0.15353799	0.00405882	0.00495169	0.00496227	0.00405125	0.00494174	0.00495119
0.16415501	0.00433948	0.00529420	0.00530546	0.00433138	0.00528352	0.00529362
0.16426897	0.00434249	0.00529788	0.00530915	0.00433439	0.00528719	0.00529730
0.16438299	0.00434551	0.00530156	0.00531284	0.00433740	0.00529086	0.00530098

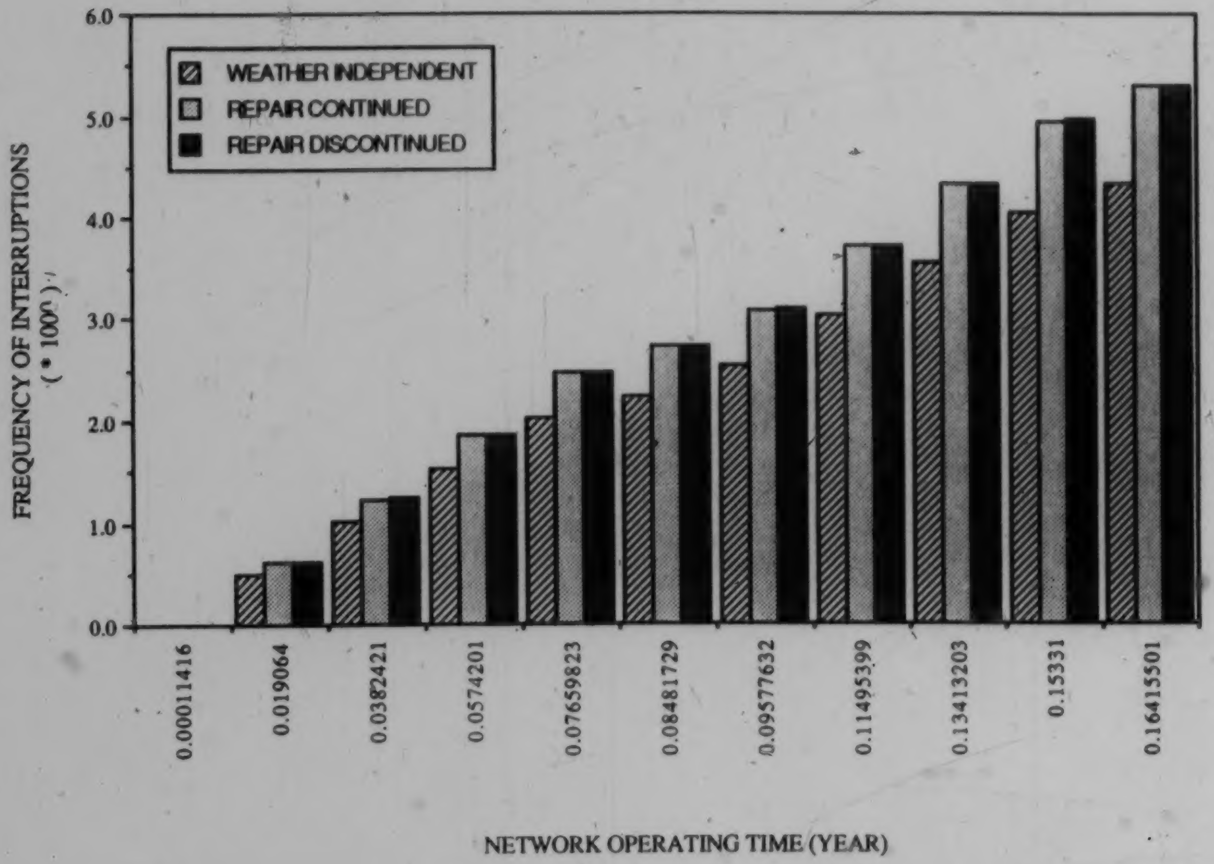


Figure C.10 Frequency of interruptions experienced by element #38 - excluding switching

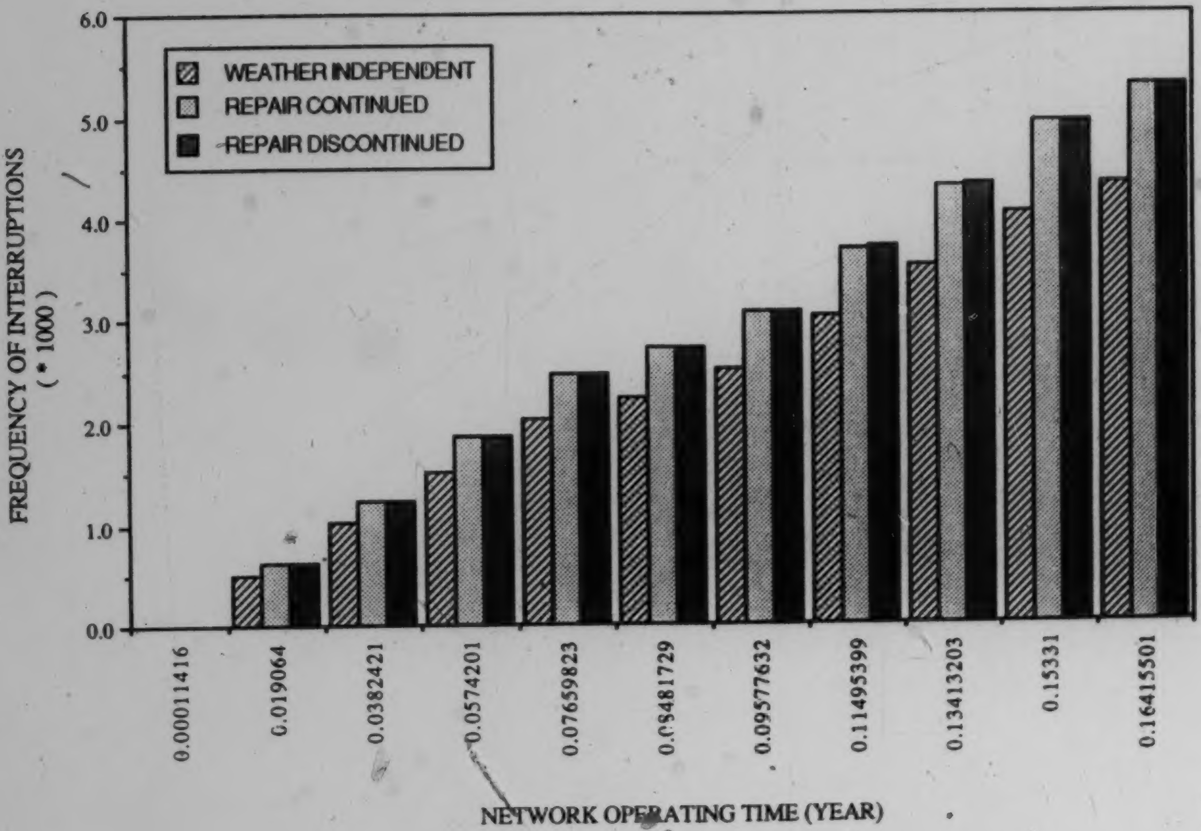


Figure C.11 Frequency of interruptions experienced by element #38 - including switching